Dawson College: Calculus II (SCIENCE): 201-NYB-05-S10: Winter 2016

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Quiz 6

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* §6.2 #62 Evaluate the integral by first completing the square.

$$\int \frac{x^2}{(3+4x-4x^2)^{3/2}}\,dx$$

$$3+4x-4x^2$$
$$= -4\left(x^2 - x - \frac{3}{4}\right)$$
$$= -4\left(x^2 - x + \frac{1}{4} - \frac{1}{4} - \frac{3}{4}\right)$$
$$= -4\left(\left(x-\frac{1}{2}\right)^2 - 1\right)$$
$$= 4 - 4\left(x-\frac{1}{2}\right)^2$$
$$= 4 - (2x-1)^2$$

$$= \int \frac{x^2}{\left(4-(2x-1)^2\right)^{3/2}}\,dx$$

$$2x-1 = 2\sin\theta$$
$$2dx = 2\cos\theta\,d\theta$$
$$dx = \cos\theta\,d\theta$$
$$x = \sin\theta + \frac{1}{2}$$

$$= \int \frac{\left(\sin\theta + \frac{1}{2}\right)^2}{\left(4-(2\sin\theta)^2\right)^{3/2}}\cos\theta\,d\theta$$

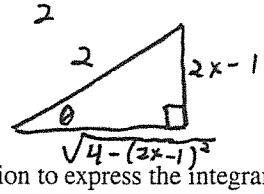
$$= \int \frac{\left(\sin^2\theta + \sin\theta + \frac{1}{4}\right)}{\left(4\cos^2\theta\right)^{3/2}}\cos\theta\,d\theta$$

$$= \frac{1}{8}\int \frac{\sin^2\theta}{\cos^2\theta} + \frac{\sin\theta}{\cos^2\theta} + \frac{1}{4}\sec^2\theta\,d\theta$$

$$= \frac{1}{8}\int \sec^2\theta - 1 + \tan\theta\sec\theta + \frac{1}{4}\sec^2\theta\,d\theta$$

$$= \frac{1}{8}\left[\frac{5}{4}\tan\theta + \sec\theta - \theta\right] + C$$

$$\frac{2x-1}{2} = \sin\theta$$



$$\arcsin\left(\frac{2x-1}{2}\right) = \theta$$

$$= \frac{1}{8}\left[\frac{5}{4}\frac{2x-1}{\sqrt{4-(2x-1)^2}} + \frac{2}{\sqrt{4-(2x-1)^2}} - \arcsin\left(\frac{2x-1}{2}\right)\right] + C$$

Question 2. *(5 marks)* §6.3 #39 Make a substitution to express the integrand as a rational function and then evaluate the integral.

$$\int \frac{e^{2x}}{e^{2x}+3e^x+2}\,dx$$

$$u = e^x$$
$$du = e^x\,dx$$

$$= \int \frac{e^x e^x}{(e^x)^2 + 3e^x + 2}\,dx$$

$$= \int \frac{u}{u^2+3u+2}\,du$$

$$= \int \frac{-1}{u+1} + \frac{2}{u+2}\,du$$

$$\frac{u}{u^2+3u+2} = \frac{u}{(u+1)(u+2)} = \frac{A_1}{u+1} + \frac{A_2}{u+2}$$

$$u = A_1(u+2) + A_2(u+1)$$

Let $u=-1 \Rightarrow A_1 = -1$

$u=-2 \Rightarrow A_2 = 2$

$$= -\ln|u+1| + 2\ln|u+2| + C$$
$$= -\ln|e^x+1| + 2\ln|e^x+2| + C$$