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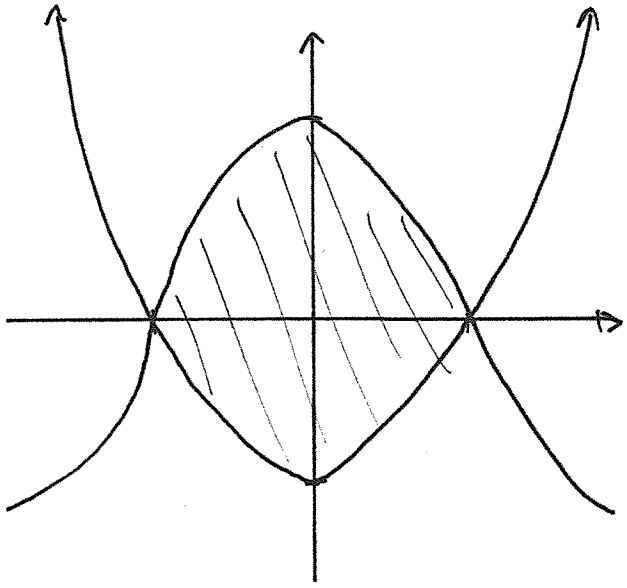
Quiz 8

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* §7.1 #17 Sketch the region enclosed by the given curves and find its area.

$y = \cos \pi x, \quad y = 4x^2 - 1$



$y = \cos(\pi(\frac{1}{2})) = 0 = \cos\left(\pi\left(-\frac{1}{2}\right)\right)$

$y = 4\left(\frac{1}{2}\right)^2 - 1 = 0 = 4\left(-\frac{1}{2}\right)^2 - 1$

Hence intersection at $x = \frac{1}{2}, -\frac{1}{2}$

$$\int_{-\frac{1}{2}}^{\frac{1}{2}} \overbrace{\cos \pi x - [4x^2 - 1]}^{f(x)}\, dx$$

since $f(-x) = f(x)$

$$= 2\int_0^{\frac{1}{2}} \cos \pi x - 4x^2 + 1\, dx$$

$$= 2\left[\frac{1}{\pi}\sin \pi x - \frac{4x^3}{3} + x\right]_0^{\frac{1}{2}}$$

$$= 2\left[\frac{1}{\pi}\sin\frac{\pi}{2} - \frac{4\left(\frac{1}{2}\right)^3}{3} + \frac{1}{2}\right] = \frac{2}{\pi} - \frac{1}{3} + \frac{1}{2} = \frac{2}{\pi} + \frac{1}{6}$$

Question 2. *(5 marks)* §7.4 #16 Find the exact length of the curve.

$y = \sqrt{x - x^2} + \arcsin(\sqrt{x})$ domain $[0,1]$

$$S = \int_a^b \sqrt{1 + (y')^2}\, dx$$

$$y' = \frac{1}{2\sqrt{x - x^2}}(1 - 2x) + \frac{1}{\sqrt{1 - (\sqrt{x})^2}} \cdot \frac{1}{2\sqrt{x}}$$

$$= \frac{1 - 2x}{2\sqrt{x}\sqrt{1 - x}} + \frac{1}{2\sqrt{x}\sqrt{1 - x}}$$

$$= \frac{2 - 2x}{2\sqrt{x}\sqrt{1 - x}}$$

$$= \frac{1 - x}{\sqrt{x}\sqrt{1 - x}}$$

$$= \int_0^1 \sqrt{1 + \left(\frac{1 - x}{\sqrt{x}\sqrt{1 - x}}\right)^2}$$

$$= \int_0^1 \sqrt{1 + \frac{(1 - x)^2}{x(1 - x)}}\, dx$$

$$= \int_0^1 \sqrt{\frac{x + 1 - x}{x}}$$

$$= \int_0^1 \frac{1}{\sqrt{x}}\, dx$$

$$= \left[2\sqrt{x}\right]_0^1 = 2$$