

Test 2

This test is graded out of 40 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. Let f be a continuous odd function and $g(x) = \int_0^x f(t) dt$.

a. (3 marks) Show that $g(x)$ is an even function.

b. (2 marks) Show that

$$\int_{-a}^a g(x)f(x) dx = 0.$$

Question 2. (5 marks) Apply partial fraction decomposition to the following rational function.

$$R(x) = \frac{x^5 + x - 1}{x^3 + 1}$$

Question 3. (5 marks) Determine whether the following integral converges. If the integral converges, determine the value to which it converges.

$$\int_0^{\pi/2} \cot^3 x \csc^3 x \, dx$$

Question 4. (5 marks) Determine whether the following integral converges. If the integral converges, determine the value to which it converges.

$$\int_1^{\infty} \frac{\arctan x}{x^2} dx$$

Question 5. (5 marks) Evaluate the integral.

$$\int \frac{1}{\sqrt{x^2 - 4x}} dx$$

Question 6. (5 marks) Sketch the region(s) enclosed by the given curves and find the total area of the enclosed region(s).

$$y = \sin \pi x, \quad y = x^2 - x, \quad x = \frac{3}{2}$$

Question 7. (5 marks) Find the length of the curve

$$y = \int_1^x \sqrt{t^2 - 1} dt, \quad 8 \leq x \leq 64$$

Question 8.¹ (5 marks) Evaluate the integral, if possible.

$$\int_0^{\pi/3} \sec x \ln(\sec x + \tan x) dx$$

¹from a John Abbott final examination

Bonus Question. (3 marks)

Integrate.

$$\int \cos \left(\lim_{n \rightarrow \infty} \sum_{i=1}^n \sin \left(i \frac{x}{n} \right) \frac{x}{n} \right) \sin x \, dx$$