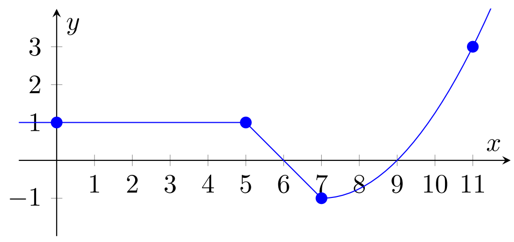
1. Let $f(t)$ be the function shown below.



[3 pt] (a) Use a Riemann sum with $n = 4$ and right end points to estimate the value of $\int_3^{11} f(t)\,dt$.

[1 pt] (b) If $f(t)$ represents the velocity of a particle at time t, find the distance travelled over the interval $[0, 6]$.

[3 pt] (c) If $\int_6^{11} \big(6f(t) - 1\big)\,dt = 0$, what is the exact value of $\int_7^{11} f(t)\,dt$?

Let $g(x) = \displaystyle\int_5^x f(t)\,dt$, again with $f(t)$ being the function in the graph above.

[3 pt] (d) Determine the values of $g(0), g(5)$, and $g(7)$.

[1 pt] (e) What are the critical values of $g(x)$?

[3 pt] (f) If $h(x) = \displaystyle\int_0^{\sin x} g(x)\,dx$, find $h''(0)$.

[5 pt] 2. Find $\int_0^3 (1 - 2x^2)\,dx$ by using the definition of the definite integral (that is, taking the limit of a Riemann sum). No points will be given for using the Fundamental Theorem of Calculus.

Summation formulas:

$$\sum_{i=1}^{n} c = cn \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

[5 pt] 3. Evaluate the limit $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} \sin\left(\pi + \frac{2\pi i}{n}\right)\frac{2\pi}{n}$ by interpreting it as a definite integral.

4. Evaluate.

[5 pt] (a) $\displaystyle\int_0^{\sqrt{3}/2} \arcsin x\,dx$

[5 pt] (b) $\displaystyle\int_1^3 \frac{2 + |x - 2|}{x}\,dx$

5. Evaluate.

[5 pt] (a) $\displaystyle\int e^{3x}\sin(2x)\,dx$

[5 pt] (b) $\displaystyle\int \frac{x^2}{\sqrt{2 - 3x}}\,dx$

6. In many population models, restrictions on resources mean that exponential growth cannot continue indefinitely. One such model is the *logistic growth model*. In this model, if the initial population is P_0 and the maximum sustainable population is K, then the population at time t can be given by

$$P(t) = \frac{K}{1 + Ae^{-ct}}$$

where c is some real constant and $A = \dfrac{K - P_0}{P_0}$.

Suppose that a colony of rabbits begins with 50 animals, the maximum sustainable population is 500 animals, and for this population $c = \frac{1}{2}$. Let $P(t)$ denote the population of the colony after t months.

[1 pt] (a) Show that $P(t) = \dfrac{500\,e^{t/2}}{e^{t/2} + 9}$.

[5 pt] (b) Find the average number of rabbits in the population over the first year (you may use part (a) even if you haven't shown it). Round your answer to the nearest rabbit.

[5 pt] 7. **[BONUS]** Let f be a continuous function and let $g(x) = \int_a^x f(t)\,dt$. Suppose that $\int_0^1 f(t)\,dt = 0$.

(a) Show that $g(0) = g(1)$.

(b) Show that

$$\int_0^1 f(x)h'(g(x))\,dx = 0$$

for *any* differentiable function h.