Dawson College: Linear Algebra (SCIENCE): 201-NYC-05-S6: Winter 2016

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Quiz 10

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. 4.4 *(5 marks)* Determine whether the given pair of lines has a point of intersection; if so, determine the scalar equation of the plane containing the lines, and if not, determine the distance between the lines.

$\mathcal{L}_1$: $\vec{x} = (1,3,1) + s\underbrace{(-2,-1,1)}_{\vec{d}_1}$ and $\mathcal{L}_2$: $\vec{x} = (0,1,7) + t\underbrace{(3,0,1)}_{\vec{d}_2}$, $s,t \in \mathbb{R}$

$\vec{d}_1$ and $\vec{d}_2$ are not $\parallel$ $\therefore$ the lines are not $\parallel$.
Lets determine if the lines intersect.

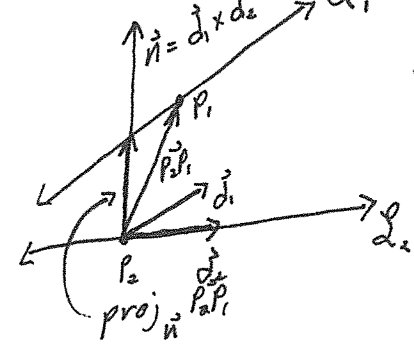
$\mathcal{L}_1$: $x = 1-2s$, $y = 3-s$, $z = 1+s$ $\qquad$ $\mathcal{L}_2$: $x = 3t$, $y = 1$, $z = 7+t$

$1-2s = 3t$ ①
$3-s = 1$ $\Rightarrow$ ② $s = 2$
$1+s = 7+t$ ③

sub ② into ①: $1-2(2) = 3t$
$-3 = 3t$
④ $-1 = t$

sub ② and ④ into ③ to check for consistency
$1+2 \overset{?}{=} 7-1$
$3 \neq 6$
$\therefore$ not consitent

$\therefore$ The two lines do not intersect
$\therefore$ The two lines are skew lines.



Shortest distance $= \|\text{proj}_{\vec{n}} \vec{P_2P_1}\|$

$\vec{n} = \vec{d}_1 \times \vec{d}_2$
$= \left(\begin{vmatrix} -1 & 0 \\ 1 & 1 \end{vmatrix}, -\begin{vmatrix} -2 & 3 \\ 1 & 1 \end{vmatrix}, \begin{vmatrix} -2 & 3 \\ -1 & 0 \end{vmatrix} \right)$
$= (-1, 5, 3)$

$\vec{P_2P_1} = P_1 - P_2$
$= (1,3,1) - (0,1,7)$
$= (1,2,-6)$

$d = \|\text{proj}_{\vec{n}} \vec{P_2P_1}\| = \left\| \dfrac{(1,2,-6)\cdot(-1,5,3)}{(-1,5,3)\cdot(-1,5,3)} (-1,5,3) \right\|$
$= \left\| \dfrac{-9}{35} (-1,5,3) \right\|$
$= \dfrac{9\sqrt{35}}{35}$

Question 2. *(5 marks)* §4.1 #2 Let V be the set of all ordered pairs of real numbers, and consider the following addition and scalar multiplication operations on $\vec{u} = (u_1, u_2)$ and $\vec{v} = (v_1, v_2)$.

$$\vec{u} + \vec{v} = (u_1 + v_1 + 1, u_2 + v_2 + 1) \qquad k\vec{u} = (ku_1, ku_2)$$

a. Compute $\vec{u} + \vec{v}$ and $k\vec{u}$ for $\vec{u} = (0,4)$ and $\vec{v} = (1,-3)$, and $k = 2$.

b. Show that $(0,0) \neq \vec{0}$.

c. Show that $(-1,-1) = \vec{0}$.

d. Show that Axiom 5 holds by producing an ordered pair $-\vec{u}$ such that $\vec{u} + (-\vec{u}) = \vec{0}$ for $\vec{u} = (u_1, u_2)$.

e. Find two vector space axioms that fail to hold.

a) $\vec{u} + \vec{v} = (0+1+1, 4-3+1) = (2,2)$
$k\vec{u} = (\ , \ )$

b,c) Let $\vec{0} = (a,b)$ and $\vec{u} = (u_1, u_2)$
$\vec{u} + \vec{0} = \vec{u}$
$(u_1 + a + 1, u_2 + b + 1) = (u_1, u_2)$
$u_1 + a + 1 = u_1$ $\qquad$ $u_2 + b + 1 = u_2$
$a = -1$ $\qquad$ $b = -1$
$\therefore \vec{0} = (a,b) = (-1,-1)$.

d) Let $\vec{w} = -\vec{u} = (w_1, w_2)$
$\vec{u} + \vec{w} = \vec{0}$
$(u_1 + w_1 + 1, u_2 + w_2 + 1) = (-1,-1)$
$u_1 + w_1 + 1 = -1$ $\qquad$ $u_2 + w_2 + 1 = -1$
$w_1 = -2 - u_1$ $\qquad$ $w_2 = -2 - u_2$
$\vec{w} = -\vec{u} = (-2-u_1, -2-u_2)$

$r(\vec{u}+\vec{v}) = r\vec{u} + r\vec{v}$ fails
since LHS $= r(u_1+v_1+1, u_2+v_2+1)$
$= (r(u_1+v_1+1), r(u_2+v_2+1))$
RHS $= (ru_1, ru_2) + (rv_1, rv_2) = (ru_1 + rv_1 + 1, ru_2 + rv_2 + 1) \neq$ LHS

$(r+s)\vec{u} = r\vec{u} + r s$ fails since
LHS $= (r+s)(u_1, u_2) = ((r+s)u_1, (r+s)u_2)$
RHS $= (ru_1, ru_2) + (sv_1, sv_2) = (ru_1 + sv_1 + 1, ru_2 + sv_2 + 1)$
$\neq$ LHS