Dawson College: Linear Algebra (SCIENCE): 201-NYC-05-S6: Winter 2016

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Quiz 11

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(2 marks)* §4.2 TF Determine whether the statement is true or false, and justify your answer.
The solution set of a consistent linear system $Ax = b$ of m equations in n unknowns is a subspace of $\mathbb{R}^n$.

False, $W = \{x \mid x \in \mathbb{R}^n \text{ and } Ax = b\}$

Let $x, y \in W$

$x+y \notin W$ since $A(x+y) = Ax + Ay = b + b = 2b$.

Question 2. *(3 marks)* §4.2 #2g Determine whether [redacted] the following are subspaces of $\mathscr{M}_{n\times n}$. The set of all $n \times n$ matrices A such that $AB = BA$ for some fixed $n \times n$ matrix B.

$W = \{A \mid A \in M_{n\times n} \text{ and } AB = BA\}$

Let $M_1, M_2 \in W$ then $M_1 + M_2 \in W$ since
$$(M_1+M_2)B = M_1B + M_2B$$
$$= BM_1 + BM_2 \quad \text{since } M_1, M_2 \in W$$
$$= B(M_1+M_2)$$

Let $M \in W$ and $r \in \mathbb{R}$ then $rM \in W$ since
$$(rM)B = r(MB) = rBM$$
$$= B(rM)$$
since $M \in W$

$\therefore$ closed under addition and scalar multiplication.

$\therefore$ by the subspace test, W is a subspace of $M_{n\times n}$

Question 3. *(5 marks)* §4.3 #15 Show that if $\{\vec{v}_1, \vec{v}_2\}$ is linearly independent and $\vec{v}_3$ does not lie in span$(\{\vec{v}_1, \vec{v}_2\})$ then $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$. is l.i.

Suppose $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is linearly dependent then $\exists c_i$ s.t. $c_i \neq 0$.

If $c_3 = 0$ then either $c_1 \neq 0$ or $c_2 \neq 0 \Rightarrow c_1\vec{v}_1 + c_2\vec{v}_2 = \vec{0}$ has a non trivial solution $\lightning$ since $\{\vec{v}_1, \vec{v}_2\}$ is linearly independent.

If $c_3 \neq 0$ then
$$c_1\vec{v}_1 + c_2\vec{v}_2 + c_3\vec{v}_3 = \vec{0}$$
$$\vec{v}_3 = -\frac{c_2}{c_3}\vec{v}_2 - \frac{c_1}{c_3}\vec{v}_1$$
$\lightning$ since $\vec{v}_3 \notin \text{span}(\{\vec{v}_1, \vec{v}_2\})$

$\therefore$ $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is linearly independent.