Dawson College: Linear Algebra (SCIENCE): 201-NYC-05-S6: Winter 2016

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Quiz 12

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(2 marks)* §4.4 TF Determine whether the statement is true or false, and justify your answer.
If $V = \text{span}\{\vec{v}_1, \ldots, \vec{v}_n\}$ then $\{\vec{v}_1, \ldots, \vec{v}_n\}$ is a basis for V.

False

$\text{span}(\{\vec{0}, \vec{i}, \vec{j}\}) = \mathbb{R}^2$ but $\{\vec{0}, \vec{i}, \vec{j}\}$ is not a basis for $\mathbb{R}^2$ since it is not linearly independent since the set contains the $\vec{0}$ vector.

Question 2. *(2 marks)* §4.4 TF Determine whether the statement is true or false, and justify your answer.
Every linearly independent subset of a vector space V is a basis V.

False

If $V = \mathbb{R}^2$ then $S = \{(1,0)\}$ is linearly independent but is not a basis for $\mathbb{R}^2$ since $(0,1) \in \mathbb{R}^2$ and $(0,1) \notin \text{span}(S)$ (does not span $\mathbb{R}^2$).

Question 3. *(2 marks)* §4.4 TF Determine whether the statement is true or false, and justify your answer.
If $\{\vec{v}_1, \ldots, \vec{v}_n\}$ is a basis for a vector space V, then every vector in V can be expressed as a linear combination of $\vec{v}_1, \ldots, \vec{v}_n$.

True,

Since $B = \{\vec{v}_1, \ldots, \vec{v}_n\}$ is a basis $V = \text{span}(B)$. Hence $\vec{v} \in V$ then $\vec{v} \in \text{span}(B)$ and $\vec{v} = c_1\vec{v}_1 + c_2\vec{v}_2 + \cdots + c_n\vec{v}_n$.

Question 4. *(4 marks)* §4.5 TF Determine whether the statement is true or false, and justify your answer.
There is a basis for $\mathscr{M}_{2\times 2}$ consisting of invertible matrices.

True, Let $S = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix} \right\}$, If S is l.i. then S also spans $M_{2\times 2}$ since S has 4 vectors and $\dim(M_{2\times 2}) = 4$. Lets verify l.i. $c_1 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + c_2 \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + c_3 \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} + c_4 \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & \text{[illegible]} & 2 & 0 \end{array}\right] \sim_{\substack{-R_2+R_3 \to R_3 \\ -R_1+R_4 \to R_4}} \left[\begin{array}{cccc|c} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & & 0 \end{array}\right]$$

$\therefore$ only trivial sol.
$\therefore$ S is l.i
$\therefore$ S is a basis since it also spans $M_{2\times 2}$

Note: the elements of S are invertible matrices