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Quiz 2

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §1.2 #3a *(3 marks)* Suppose that the augmented matrix for a system of linear equations has been reduced by row operations to the given row echelon form. Solve the system.

$$\begin{matrix} x_1 & x_2 & x_3 & x_4 \end{matrix}\begin{bmatrix} 1 & 0 & 8 & -5 & 6 \\ 0 & 1 & 4 & -9 & 3 \\ 0 & 0 & 1 & 1 & 2 \end{bmatrix}$$

Let $x_4 = t$ where $t \in \mathbb{R}$

sub into

$$\begin{cases} x_1 + 8x_3 - 5x_4 = 6 \\ x_2 + 4x_3 - 9x_4 = 3 \\ x_3 + x_4 = 2 \end{cases}$$

$$\begin{cases} x_1 + 8x_3 - 5t = 6 \quad (1) \\ x_2 + 4x_3 - 9t = 3 \quad (2) \\ x_3 + t = 2 \quad (3) \end{cases}$$

From (3) $x_3 = 2 - t$ sub into (2)

$x_2 + 4(2-t) - 9t = 3$

$x_2 = -5 + 13t$

sub x_2, x_3 into (1)

$x_1 + 8(2-t) - 5t = 6$

$x_1 = 13t - 10$

$\therefore (x_1, x_2, x_3, x_4)$

$= (13t-10, 13t-5, 2-t, t) \quad t \in \mathbb{R}$

Question 2. §1.2 #8 *(4 marks)* Solve the given linear system by Gauss-Jordan elimination.

$$\begin{aligned} -2b + 3c &= 1 \\ 3a + 6b - 3c &= -2 \\ 6a + 6b + 3c &= 5 \end{aligned}$$

$$\begin{bmatrix} 0 & -2 & 3 & 1 \\ 3 & 6 & -3 & -2 \\ 6 & 6 & 3 & 5 \end{bmatrix}$$

$\sim R_1 \leftrightarrow R_2$

$$\begin{bmatrix} 3 & 6 & -3 & -2 \\ 0 & -2 & 3 & 1 \\ 6 & 6 & 3 & 5 \end{bmatrix}$$

$\sim -2R_1 + R_3 \to R_3$

$$\begin{bmatrix} 3 & 6 & -3 & -2 \\ 0 & -2 & 3 & 1 \\ 0 & -6 & 9 & 9 \end{bmatrix}$$

$\sim -3R_2 + R_3 \to R_3$

$$\begin{bmatrix} 3 & 6 & -3 & -2 \\ 0 & -2 & 3 & 1 \\ 0 & 0 & 0 & 6 \end{bmatrix}$$

$0a + 0b + 0c = 6$

$0 = 6$

has no solution.

Hence the system has no solution.

Question 3. §1.2 #TF *(3 marks)* Determine whether the statement is true or false, and justify your answer.
If a linear system has more unknowns than equations, then it must have infinitely many solutions.

False, the system associated to the augmented matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

has 3 unknowns but has no solutions since $0x + 0y + 0z = 1$ has no solution.