Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S6: Winter 2016

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Quiz 3

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §1.3 #15 *(4 marks)* Find all values of k, if any, that satisfy the equation.

$$\begin{bmatrix} k & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & 2 & -3 \end{bmatrix} \begin{bmatrix} k \\ 1 \\ 1 \end{bmatrix} = 0$$

$$\begin{bmatrix} K & 1 & 1 \end{bmatrix} \begin{bmatrix} K+1 \\ K+2 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$$

$$\begin{bmatrix} K(K+1) + K+2 - 1 \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$$

$$\begin{bmatrix} K^2+K + K+2-1 \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$$

$$\begin{bmatrix} K^2 + 2K + 1 \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$$

$$K^2+2K+1 = 0$$
$$(K+1)^2 = 0$$
$$K = -1$$

Question 2. §1.3 #30b *(2 marks)* Let 0 denote a 2×2 matrix, each of whose entries is zero. Is there a 2×2 matrix A such that $A \neq 0$ and $AA = A$? Justify your answer.

Yes, $A = I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$AA = II = I = A.$

Question 3. §1.3 #TF *(2 marks)* Determine whether the statement is true or false, and justify your answer.
For every square matrix A, it is true that $tr(A^T) = tr(A)$.

True, Let $A = [a_{ij}]_{n\times n}$ and $A^T = [a_{ji}]_{n\times n}$

$tr(A^T) = tr([a_{ji}]) = a_{11} + a_{22} + \dots + a_{nn} = tr(A)$

Question 4. §1.3 #TF *(2 marks)* Determine whether the statement is true or false, and justify your answer.
If A is an $n \times n$ matrix and c is a scalar, then $tr(cA) = ctr(A)$.

True, Let $A = [a_{ij}]_{n\times n}$

$$\begin{aligned} tr(cA) = tr(c[a_{ij}]_{n\times n}) = tr([ca_{ij}]_{n\times n}) &= ca_{11} + ca_{22} + \dots + ca_{nn} \\ &= c(a_{11} + a_{22} + \dots + a_{nn}) \\ &= c\,tr(A) \end{aligned}$$