Dawson College: Linear Algebra (SCIENCE): 201-NYC-05-S6: Winter 2016

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Quiz 4

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §1.5 #27 *(4 marks)* Find all values of c, if any, for which the given matrix is invertible.

$\begin{bmatrix} c & c & c \\ 1 & c & c \\ 1 & 1 & c \end{bmatrix}$ If $c=0$ then R_1 is a row of zero so the RREF will not be I $\therefore$ To be invertible $c \neq 0$.

If $c \neq 0$ $\sim_{\frac{1}{c}R_1 \to R_1} \begin{bmatrix} 1 & 1 & 1 \\ 1 & c & c \\ 1 & 1 & c \end{bmatrix} \sim_{\substack{-R_1+R_2 \to R_2 \\ -R_1+R_3 \to R_3}} \begin{bmatrix} 1 & 1 & 1 \\ 0 & c-1 & c-1 \\ 0 & 0 & c-1 \end{bmatrix}$

If $c \neq 1$ then the RREF is I so the matrix is invertible by TFAE.

$\therefore$ the matrix is invertible iff $c \neq 0$ and $c \neq 1$

Question 2. §1.5 #TF *(2 marks)* Determine whether the statement is true or false, and justify your answer.
If A is an $n \times n$ matrix that is not invertible, then the matrix obtained by intechanging two rows of A cannot be invertible.

True, $A \sim_{\substack{\text{Gauss Jordan} \\ k \text{ elem row op}}} \sim R \neq I$ since A is not invertible by TFAE

Using the k elem. row op. we get k elem. matrices s.t. $E_k \cdots E_2 E_1 A = R$

Let B be the matrix obtained by interchanging 2 rows of A.

It follows $\exists E$ s.t. $EA = B$ and $A = E^{-1}B$.

So $E_k \cdots E_2 E_1 E^{-1} B = R \neq I$ Hence the RREF of B is the same as the RREF of A. So B is not invertible since $R \neq I$.

Question 3. §1.4 #TF *(2 marks)* Determine whether the statement is true or false, and justify your answer.
A square matrix containing a row or a column of zeros cannot be invertible.

True, Suppose the i^{th} row of A is a row of zeros. And B is any matrix (same size as A). Then the i^{th} row of the product AB is $[0\ 0 \dots 0]B = [0\ 0 \dots 0]$

Hence impossible to find a matrix B s.t. $AB = I$.
Similar argument for column.

Question 4. §1.4 #TF *(2 marks)* Determine whether the statement is true or false, and justify your answer.
The sum of two invertible matrices of the same size must be invertible.

False, $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ are both invertible matrices but $A+B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ is not invertible.