Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S6: Winter 2016

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Quiz 5

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §1.7 #TF *(2 marks)* Determine whether the statement is true or false, and justify your answer.
If A and B are $n \times n$ matrices such that $A+B$ is symmetric, then A and B are symmetric.

False $A+B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ is symmetric

but A and B are not symmetric

Question 2. §1.7 #TF *(2 marks)* Determine whether the statement is true or false, and justify your answer.
If A^2 is a symmetric matrix, then A is a symmetric matrix.

False,

$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ is not symmetric but

$A^2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ is symmetric

Question 3. §1.6 #TF *(2 marks)* Determine whether the statement is true or false, and justify your answer.
If A and B are row equivalent matrices, then the linear systems $Ax = 0$ and $Bx = 0$ have the same solution set.

True, since A and B have the same RREF. $\therefore$ same solution set.

A and B have the same RREF since

$A \sim_{k \text{ elem row op}} B \sim_{\text{Gauss Jordan } \ell \text{ elem row op.}} R$

from row equivalence.

Question 4. §1.6 #TF *(2 marks)* Determine whether the statement is true or false, and justify your answer.
If A is an $n \times n$ matrix and S is an $n \times n$ invertible matrix, then if x is a solution to the linear system $(S^{-1}AS)x = b$, then Sx is a solution to the linear system $Ay = Sb$.

True,

since x is a solution to $S^{-1}ASx = b$

$SS^{-1}ASx = Sb$

$ASx = Sb$

So $y = Sx$ is a solution to $Ay = Sb$.

Question 5. §1.6 #TF *(2 marks)* Determine whether the statement is true or false, and justify your answer.
Let A be an $n \times n$ matrix. The linear system $Ax = 4x$ has a unique solution if and only if $A - 4I$ is an invertible matrix.

True,

[$\Rightarrow$] $Ax = 4x$

$Ax - 4x = 0$

$(A - 4I)x = 0$

$A - 4I$ invertible by TFAE since the above has a unique solution

[$\Leftarrow$] $A - 4I$ is invertible, so by TFAE $(A-4I)x = 0$ has a unique solution.

$Ax - 4x = 0$

$Ax = 4x$ has a unique solution.