

## Quiz 6

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

**Question 1.** §2.1 #25 (5 marks) Evaluate  $\det(A)$  by a cofactor expansion along a row or column of your choice.

$$\begin{aligned}
 A &= \begin{bmatrix} 3 & 3 & 0 & 5 \\ 2 & 2 & 0 & -2 \\ 4 & 1 & -3 & 0 \\ 2 & 10 & 3 & 2 \end{bmatrix} \quad |A| = 0C_{13} + 0C_{23} + (-3)C_{33} + 3C_{43} \\
 &= -3(-1)^{7+3} \begin{vmatrix} 3 & 3 & 5 \\ 2 & 2 & -2 \\ 2 & 10 & 2 \end{vmatrix} + 3(-1)^{4+3} \begin{vmatrix} 3 & 3 & 5 \\ 2 & 2 & -2 \\ 4 & 1 & 0 \end{vmatrix} \\
 &= -3 \left[ 3 \begin{vmatrix} 2 & -2 \\ 10 & 2 \end{vmatrix} - 3 \begin{vmatrix} 2 & -2 \\ 2 & 2 \end{vmatrix} + 5 \begin{vmatrix} 2 & 2 \\ 2 & 10 \end{vmatrix} \right] - 3 \left[ 3 \begin{vmatrix} 2 & -2 \\ 1 & 0 \end{vmatrix} - 3 \begin{vmatrix} 2 & -2 \\ 4 & 0 \end{vmatrix} \right. \\
 &\qquad \qquad \qquad \left. + 5 \begin{vmatrix} 2 & 2 \\ 4 & 1 \end{vmatrix} \right] \\
 &= -3 \left[ 3(4+20) - 3(4+4) + 5(20-4) \right] \\
 &\quad - 3 \left[ 3(0+2) - 3(0+8) + 5(2-8) \right] \\
 &= -3 \left[ 3(4) + 3(20) - 3(4) - 3(4) + 5(20) - 4(5) + 3(2) - 3(8) + 5(2) - 5(8) \right] \\
 &= -3 \left[ 60 - 12 + 100 - 20 + 6 - 24 + 10 - 40 \right] = -3 \left[ 80 \right] = -240.
 \end{aligned}$$

**Question 2.** §2.2 #29 (5 marks) Use row reduction to show that

$$\det \begin{pmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{pmatrix} = (b-a)(c-a)(c-b)$$

$$\begin{bmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{bmatrix}$$

$$\begin{aligned}
 \det &= 1 \cdot (b-a) [(c-a)(c+a) - (b+a)(c-a)] \\
 &= (b-a)(c-a) [c+a - (b+a)] \\
 &= (b-a)(c-a)(c-b)
 \end{aligned}$$

$$\sim \begin{matrix} -aR_1 + R_2 \rightarrow R_2 \\ -a^2R_1 + R_3 \rightarrow R_3 \end{matrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & b-a & c-a \\ 0 & b^2-a^2 & c^2-a^2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 0 & b-a & c-a \\ 0 & (b-a)(b+a) & (c-a)(c+a) \end{bmatrix}$$

$$\sim \begin{matrix} -(b+a)R_2 + R_3 \rightarrow R_3 \end{matrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & b-a & c-a \\ 0 & 0 & (c-a)(c+a) - (b+a)(c-a) \end{bmatrix}$$