Name: Y. Lamontagne

Quiz 8

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §3.2 #TF *(2 marks)* Determine whether the statement is true or false, and justify your answer.
For all vectors $\vec{v}$, $\vec{v}$, and $\vec{w}$ in $\mathbb{R}^n$, we have $||\vec{u}+\vec{v}+\vec{w}|| \leq ||\vec{u}|| + ||\vec{v}|| + ||\vec{w}||$.

True, $\|\vec{u}+\vec{v}+\vec{w}\| \leq \|\vec{u}+\vec{v}\| + \|\vec{w}\|$ by the triangle inequality,
$\leq \|\vec{u}\| + \|\vec{v}\| + \|\vec{w}\|$ again by the triangle inequality.

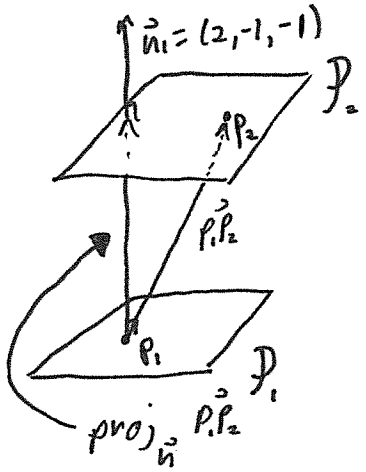
Question 2. §3.2 #TF *(2 marks)* Determine whether the statement is true or false, and justify your answer.
If $\vec{u} \cdot \vec{v} = 0$, then either $\vec{u} = \vec{0}$ or $\vec{v} = \vec{0}$.

False Let $\vec{u} = (1,0)$ and $\vec{v} = (0,1)$
$\vec{u} \cdot \vec{v} = 1(0) + 0(1) = 0$ but $\vec{u} \neq 0$ and $\vec{v} \neq 0$

Question 3. §3.3 #TF *(2 marks)* Determine whether the statement is true or false, and justify your answer.
If the relationship $\text{proj}_{\vec{a}}\vec{u} = \text{proj}_{\vec{a}}\vec{v}$ holds for some nonzero vector $\vec{a}$, then $\vec{u} = \vec{v}$.

False, Let $\vec{a} = (1,0)$, $\vec{u} = (1,1)$ and $\vec{v} = (1,-1)$
$\text{proj}_{\vec{a}}\vec{u} = (1,0)$
$\text{proj}_{\vec{a}}\vec{v} = (1,0)$ but $\vec{u} \neq \vec{v}$

Question 4. §3.3 #37 *(4 marks)* Find the distance between the given parallel planes: $2x - y - z = 5$ and $-4x + 2y + 2z = 12$



Let $x = z = 0$ then $y = -5$ $\mathcal{P}_1$ $\mathcal{P}_2$
$\therefore P_1(0,-5,0)$
Let $x = z = 0$ $\therefore P_2(0,6,0)$
So $\vec{P_1P_2} = P_2 - P_1 = (0,6,0) - (0,-5,0) = (0,11,0)$

$$\text{proj}_{\vec{n}_1}\vec{P_1P_2} = \frac{\vec{n}_1 \cdot \vec{P_1P_2}}{\vec{n}_1 \cdot \vec{n}_1}\vec{n}_1 = \frac{(2,-1,-1)\cdot(0,11,0)}{(2,-1,-1)\cdot(2,-1,-1)}(2,-1,-1)$$
$$= \frac{-11}{6}(2,-1,-1)$$

$d = \|\text{proj}_{\vec{n}}\vec{P_1P_2}\|$

$$d = \|\text{proj}_{\vec{n}_1}\vec{P_1P_2}\| = \left\|\frac{-11}{6}(2,-1,-1)\right\| = \frac{11}{6}\|(2,-1,-1)\| = \frac{11}{6}\sqrt{6}$$