Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S6: Winter 2016

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Quiz 9

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. 4.7 #TF *(3 marks)* Determine a vector equation of the line of intersection of the given planes.

$x+3y-z=5$ and $2x-5y+z=7$

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 2 & -5 & 1 & 7 \end{bmatrix} \sim_{-2R_1+R_2\to R_2} \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & -11 & 3 & -3 \end{bmatrix} \sim_{-\frac{1}{11}R_2\to R_2} \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -\frac{3}{11} & \frac{3}{11} \end{bmatrix}$$

$$\sim_{-3R_2+R_1\to R_1} \begin{bmatrix} 1 & 0 & -\frac{2}{11} & 46/11 \\ 0 & 1 & -\frac{3}{11} & \frac{3}{11} \end{bmatrix}$$

Let $z=t$

$x = \frac{46}{11} + \frac{2}{11}t$

$y = \frac{3}{11} + \frac{3}{11}t$

$\therefore \vec{x} = (x,y,z) = (\frac{46}{11}, \frac{3}{11}, 0) + t(\frac{2}{11}, \frac{3}{11}, 1)$

Question 2. §3.4 #TF *(2 marks)* Determine whether the statement is true or false, and justify your answer.
The general solution of the nonhomogeneous linear system $Ax = b$ can be obtained by adding b to the general solution of the homogeneous linear system $Ax = 0$.

False, Given $\underbrace{\begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}}_{A}\underbrace{\begin{bmatrix} x \\ y \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{b}$, The solution set of $Ax=0$: $\left[\begin{array}{cc|c} 2 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right]$

is $(x,y) = (0,t)$

$t \in \mathbb{R}$

and $(x,y) = b + (0,t)$
$= (1,0) + (0,t)$
is not the solution set of $Ax=b$

Question 3. §3.5 #TF *(5 marks)* Prove: If $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$ lie in the same plane, then

$(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d}) = \vec{0}$

Suppose $\vec{a} \parallel \vec{b}$ then $\exists k$ s.t $\vec{a} = k\vec{b}$ and $(\vec{a}\times\vec{b}) = (k\vec{b}\times\vec{b})$
$= k(\vec{b}\times\vec{b}) = k\vec{0}$
$= \vec{0}$ it follows that
$(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d}) = \vec{0}$

similarly if $\vec{c} \parallel \vec{d}$.

Suppose that $\vec{a}$ is not $\parallel$ to $\vec{b}$ and $\vec{c}$ is not $\parallel$ $\vec{d}$ then

$\vec{n}_1 = \vec{a}\times\vec{b}$ is orthogonal to $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ since they lie on the same plane

$\vec{n}_2 = \vec{c}\times\vec{d}$ " " " " " " " " " " " " "

Hence $\vec{n}_1 \parallel \vec{n}_2$ then $\exists K$ s.t $\vec{n}_1 = K\vec{n}_2$ and $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})$
$= \vec{n}_1 \times \vec{n}_2$
$= (K\vec{n}_2)\times\vec{n}_2 = K(\vec{n}_2\times\vec{n}_2) = K\vec{0} = \vec{0}$