Dawson College: **Linear Algebra**: 201-NYC-05-S06: Winter 2016

Name: ____________________

Test 1

This test is graded out of 40 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1.

a. *(8 marks)* Solve for all $a,\ b,\ c,\ d$ such that

$$\begin{bmatrix} 3a+3b+7c-3d & 2a+3b+3c+d \\ 4a+17c-2d & 9a+6b+27c-4d \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$$

b. *(1 mark)* Find two particular solution $(a,\ b,\ c,\ d)$ that satisfy the above matrix equation.

c. *(1 mark)* Find a solution to the above matrix equation where $c = 1$.

Question 2. Consider the matrices:

$$A = \left[a_{ij}\right]_{3\times3}, \quad B = \left[b_{ij}\right]_{2\times3}, \quad C = \left[c_{ij}\right]_{3\times2}, \quad D = \left[d_{ij}\right]_{2\times2}, \quad E = \left[e_{ij}\right]_{3\times6}$$

where $a_{ij} = i - j, \quad b_{ij} = (-1)^i 2 + (-1)^j 3, \quad c_{ij} = i + j, \quad d_{ij} = (ij)^2, \quad e_{ij} = i + j$. Evaluate the following if possible, justify.

a. *(2 marks)* EA

b. *(2 marks)* AB^T

c. *(2 marks)* $C^T AB^T$

d. *(2 marks)* $\text{trace}(D^2)$

e. *(2 marks)* $(3BC - 2D)^T$

Question 3. Consider

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 2 \\ 0 & 2 & 1 \end{bmatrix}.$$

a. *(5 marks)* Determine A^{-1}.

b. *(2 marks)* Determine $((2A)^T)^{-1}$, if possible.

Question 4. *(2 marks)* Find a formula for trace(A^{-1}) where $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ if A is invertible.

Question 5. *(3 marks)* Prove: If E_i are elementary matrices and $E_n \cdots E_2 E_1 A$ is invertible then A is invertible.

Question 6. *(3 marks)* Given a matrix A which satisfy $A^3 + 3A^2 + A + I = 0$ find the inverse of A in terms of A and the identity.

Question 7. *(5 marks)* Given

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 \end{bmatrix}, \quad E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad E_3 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

and

$$(X^T E_1 E_2 E_3)^T = A$$

solve for X, if possible.

Bonus Question. *(5 marks)*
Given

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 1 & 1 & 2 \\ 1 & 0 & 2 \end{bmatrix}$$

where the entries of the matrix are elements of $\mathbb{Z}_3$. Operations on the elements of $\mathbb{Z}_3$ can be defined by the following Cayley tables:

+	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

×	0	1	2
0	0	0	0
1	0	1	2
2	0	2	1

Express A as a product of elementary matrices, if possible.