**Name:** ____________________

# Test 2

This test is graded out of 40 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

**Question 1.** Given

$$A = \begin{bmatrix} 1 & 0 & 2 & -3 \\ 0 & -4 & 5 & 6 \\ -7 & 0 & 0 & 0 \\ 8 & 10 & 11 & 0 \end{bmatrix}.$$

a. *(5 marks)* Evaluate $\det(A)$.

b. *(3 marks)* If $B$ is a $4 \times 4$ matrix and $\det(2BA + B\text{adj}(A)A^2) = 0$ then $B$ singular.

**Question 2.** (2 marks) Prove or disprove: If $\det(A^2) = \det(A)$ then $A^2 = A$.

**Question 3.** *(3 marks)* Prove: If $A$ is an $n \times n$ skew-symmetric matrix where $n$ is odd then $\det(A) = 0$.

**Question 4.** *(2 marks)* Show that there exist an $n \times n$ symmetric elementary matrix $E$ such that $E^2 = I$.

**Question 5.** *(3 marks)* Prove or disprove: If a system of linear equations $Ax = b$ has $n$ equations and $n$ variables has infinitely many solutions then $Ax = c$ has infinitely many solutions for all $n \times 1$ matrix $c$.

**Question 6.** *(5 marks)* Given

$$A = \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 0 & d \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & 0 & \frac{3}{a} & \frac{3}{a} \\ 0 & 0 & 0 & \frac{2}{b} \\ 0 & \frac{1}{c} & 0 & \frac{1}{c} \\ \frac{4}{d} & 0 & 0 & \frac{4}{d} \end{bmatrix}$$

If $\det(B) = 5$ then determine $\det(A)$.

**Question 7.** Given $\vec{u}, \vec{v} \in \mathbb{R}^2$ where $\vec{u} = (1,1)$, $||\vec{v}|| = 1$ and the angle between $\vec{u}$ and $\vec{v}$ is $\pi/4$.

a. *(2 marks)* Determine $\vec{u} \cdot \vec{v}$.
b. *(3 marks)* Determine $\vec{v}$.

**Question 8.** Given $\mathscr{E}: \; x - 2y = 3$ and $P = (-1,\ 1)$.

a. *(5 marks)* Find the closest point on $\mathscr{E}$ from $P$.
b. *(2 marks)* Find the shortest distance from $P$ to $\mathscr{E}$.

**Question 9.** Given $\mathscr{E}:\ x-2y=3$ and $P=(-1,\ 1,\ -1)$.

a. *(2 marks)* Find the equation of a line orthogonal to $\mathscr{E}$ that passes through $P$.

b. *(2 marks)* Find the closest point on $\mathscr{E}$ from $P$.

c. *(2 marks)* Find the shortest distance from $P$ to $\mathscr{E}$.

**Bonus Question.** *(5 marks)*
Given

$$A=\begin{bmatrix}10&1&1&1&9&3&4&1&0&9\\0&9&1&1&4&9&2&7&7&9\\0&0&8&1&1&4&9&2&7&7\\0&0&0&7&1&1&4&9&2&7\\0&0&0&0&6&1&1&4&9&2\\0&0&0&0&0&5&1&1&4&9\\0&0&0&0&0&0&4&1&1&4\\0&0&0&0&0&0&0&3&1&1\\0&0&0&0&0&0&0&0&2&1\\0&0&0&0&0&0&0&0&0&1\end{bmatrix}$$

determine

$$\operatorname{adj}(\operatorname{adj}(A))$$