Name: ____________________

Test 3

This test is graded out of 38 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. Given the vectors $\vec{u} = (\lambda+1, 1, \lambda)$, $\vec{v} = (\lambda, 2, 2)$ and $\vec{w} = (1, 1, \lambda)$.

a. *(3 marks)* For which value(s) of λ if any, are the vectors $\vec{u}, \vec{v}$ and $\vec{w}$ linearly independent.

b. *(1 mark)* For which value(s) of λ if any, $\operatorname{span}(\{\vec{u}, \vec{v}, \vec{w}\}) = \mathbb{R}^3$.

c. *(2 marks)* For which value(s) of λ if any, $\operatorname{span}(\{\vec{u}, \vec{v}, \vec{w}\})$ is a plane through the origin.

d. *(2 marks)* For which value(s) of λ if any, $\operatorname{span}(\{\vec{u}, \vec{v}, \vec{w}\})$ is a line through the origin.

e. *(3 marks)* For which value(s) of λ if any, the vectors $\vec{u}, \vec{v}$ and $\vec{w}$ generate a parallelpiped of volume 2016.

Question 2. (3 marks) Determine whether the statement is true or false, and justify your answer.
The general solution of the nonhomogeneous linear system $Ax = b$ can be obtained by adding b to the general solution of the homogeneous linear system $Ax = 0$.

Question 3. *(6 marks)* Given the lines: $\mathscr{L}_1 : \vec{x} = (1,2,0) + t(-1,0,1)$, $t \in \mathbb{R}$ and $\mathscr{L}_2 : \vec{x} = (1,3,-1) + s(2,1,0)$, $s \in \mathbb{R}$. Find the shortest distance between $\mathscr{L}_1$ and $\mathscr{L}_2$.

Question 4. *(4 marks)* Determine whether

$$W = \{f : \mathbb{R} \to \mathbb{R} \mid f(-x) = -f(x)\}$$

is a subspace of the vector space of real functions.

Question 5. *(2 marks)* Determine whether the following is a vector space: $V = \{A \mid A \in \mathscr{M}_{2\times 2} \text{ and } A^T = A\}$ with the following operations:

$$A + B = AB \text{ and } kA = kA$$

That is, vector addition is matrix multiplication and scalar multiplication is the regular scalar multiplication. Justify.

Question 6. Given the set

$$S = \{1 + x + x^2, 1 + x^3\}$$

a. *(2 marks)* Determine whether S is linearly independent.

b. *(4 marks)* Find a new set S' that contains the elements of S and is a basis for $\mathscr{P}_3$. Show that S' is a basis.

c. *(2 marks)* Express $p(x) = x + x^2 - x^3$ relative to the basis S'.

Question 7. Given the set

$$S = \left\{ \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \right\}$$

a. *(4 marks)* Determine whether S is a basis for the vector space of 2×2 upper triangular matrices with the usual matrix addition and scalar multiplication.

b. *(1 mark)* Determine the dimension of the vectors space of 2×2 upper triangular matrices with the usual matrix addition and scalar multiplication.

Bonus Question. *(5 marks)*
Prove: A subspace of a finite-dimensional vector space is finite-dimensinal.