Test 2 (April 5, 2017) NAME: SOLUTIONS

Statistics for Social Science (201-401-DW)
Instructor: Emilie Richer

Instructions:

- Show all your work. Some solutions will require more written explanation than others. If you use your calculator to compute the mean and SD you do not have to show your work.
- The test is comprised of 8 questions and marked out of a total of **40 marks.**

[QUESTION 1] (5 marks)

Suppose that you are taking a quiz of 5 multiple-choice questions, each question having 4 possible responses (one correct, three incorrect). You did not study at all for the quiz and will randomly guess the correct response for each question. We define the random variable X to be the number of correct responses on the quiz.

a. Explain why this can be considered a binomial experiment.

WE HAVE A REPEATED IDENTICAL EXPERIMENT (ANSWERING QUESTIONS) WITH ONLY TWO POSSIBLE OUTCOMES: SUCCESS OR FAILURE (ANSWERING RIGHT OR WRONG)

b. State the values of n and p.

$n = 5 \quad p = 0.25 \quad q = 0.75$

c. Calculate the probability that you will pass the quiz.

$$P(x=3) + P(x=4) + P(x=5)$$
$$= \binom{5}{3}(0.25)^3(0.75)^2 + \binom{5}{4}(0.25)^4(0.75) + \binom{5}{5}(0.25)^5$$
$$= 0.0879 + 0.01465 + 0.0009766$$
$$= \boxed{0.1035}$$

d. Compute the mean, variance and standard deviation of X.

$$\mu = np = 5(0.25) = \boxed{1.25}$$

$$\sigma = \sqrt{npq} = \sqrt{5(0.25)(0.75)} = \boxed{0.968}$$

$$\sigma^2 = \boxed{0.9375}$$

[QUESTION 2] (5 marks)
The time it takes a statistician to walk to work each day is normally distributed with a mean of 20 minutes and a standard deviation of 2.5 minutes.

(a) If she leaves for work at 7:35am, what is the probability that she will arrive to work by 8:00am?

(b) At what time must she leave for work, to give herself a 90% chance of arriving by 8:00am?

(a) $P(X \leq 25)$

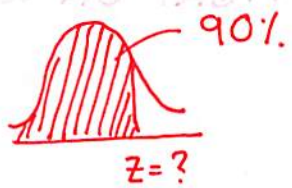
$= P\left(Z \leq \frac{25-20}{2.5}\right)$

$= P(Z \leq 2)$

$= 0.9772$

SHE HAS A 97.72% CHANCE OF ARRIVING TO WORK ON TIME

(b)



$z = 1.28$

$1.28 = \frac{x-20}{2.5}$

$3.2 = x - 20$

$x = 23.2$

SHE WOULD HAVE TO LEAVE WORK AT 7:36 AM 48 SECONDS

[QUESTION 3] (5 marks)
Statistics final grades for a particular class are normally distributed with a mean of 70 and a standard deviation of 10.

(a) What percentage of students passed the class?

(b) If we select a student at random from among those who passed the class, what is the probability that the selected student has a grade of 90 or higher?

(a) $P(X \geq 60)$

$= P\left(Z \geq \frac{60-70}{10}\right)$

$= P(Z \geq -1)$

$= 1 - 0.1587$

$= 0.8413$ $\boxed{84.13\%}$ OF STUDENTS PASSED

(b) $P(X \geq 90) = P\left(Z \geq \frac{90-70}{10}\right)$

$= P(Z \geq 2)$

$= 1 - 0.9772$

$= 0.0228$

P(90 or Higher | STUDENTS WHO PASSED)

$= \frac{0.0228}{0.8413} = \boxed{0.0271}$

[QUESTION 4] (5 marks)
A study estimates that 70% of all Montrealers are bilingual. If this is true, then what is the probability that a random sample of 2100 Montrealers will include between 1428 and 112 Montrealers, inclusively, who are bilingual?

We have a Binomial distribution
with $n = 2100$
$p = 0.7$
& we want $P(112 \leq X \leq 1428)$

We will use the normal approximation to the Binomial because the computation is too heavy.

We can do this since $np > 5$
& $nq > 5$

We use a Normal distribution with
$\mu = np = 2100(0.7) = 1470$
& $\sigma = \sqrt{npq} = \sqrt{2100(0.7)(0.3)} = 21$

We must compute $P(111.5 \leq X \leq 1428.5)$

$= P\left(\frac{111.5 - 1470}{21} \leq x \leq \frac{1428.5 - 1470}{21}\right)$

$= P(-64.7 \leq x \leq -1.98)$

$= 0.0239 - 0 = \boxed{0.0239}$

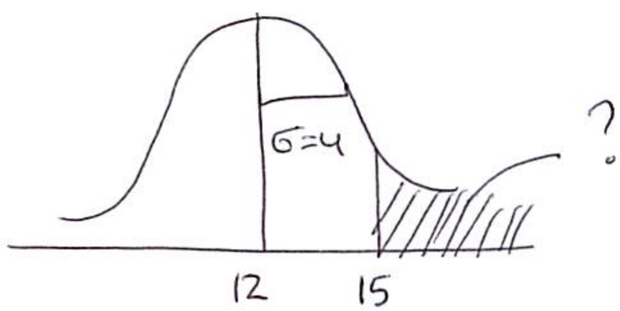
There is a 2.39% chance of sampling between 112 & 1428 bilingual Montrealers

[QUESTION 5] (5 marks)
Bell Canada reports that long-distance telephone calls in Canada are normally distributed with an average duration of 12.0 minutes and a standard deviation of 4.0 minutes.

(a) What proportion of long-distance telephone calls in Canada last longer than 15 minutes?

(b) What is the probability that the average of a random sample of 100 long-distance calls in Canada will exceed 13 minutes?

(a)



$$P(x>15) = P\left(Z > \frac{15-12}{4}\right)$$
$$= P(Z > 0.75)$$
$$= 0.2266$$

(b) WE CAN ASSUME $\bar{X}$ are Normally DISTRIBUTED b/c P_x is NORMAL (or $n > 30$)

$$P(\bar{X} > 13) = P\left(Z > \frac{13-12}{4/\sqrt{100}}\right)$$
$$= P\left(Z > \frac{1}{0.4}\right) = P(Z > 2.5)$$
$$= \underline{0.0062}$$

[QUESTION 6] (5 marks)
In the discrete probability distribution below, find μ and σ.

x	-2	1	2	4
P(x)	0.1	0.4	0.3	0.2

$$\mu = \sum x P(x)$$
$$= -2(0.1) + 1(0.4) + 2(0.3) + 4(0.2)$$
$$= -0.2 + 0.4 + 0.6 + 0.8$$
$$= 1.6$$

$$\sigma = \sqrt{x^2 P(x) - \mu^2}$$
$$= \sqrt{[4(0.1) + 1(0.4) + 4(0.3) + 16(0.2)] - (1.6)^2} = \sqrt{5.2 - (1.6)^2}$$
$$= 1.62$$

[QUESTION 7] (5 marks)
Using the standard normal distribution N(z;0,1) table to find the following probabilities:
(a) P(0 < z < 0.25)
(b) P(-1.32 < z < 0.12)
(c) P(z > -1.06)
(d) P(-1.32 < z < 0.22)
(e) P(z < -2.21)

(a) $0.5987 - 0.5 = \underline{0.0987}$

(b) $0.5478 - 0.0934 = \underline{0.4544}$

(c) $\underline{0.8554}$

(d) $0.5871 - 0.0934 = \underline{0.4937}$

(e) $\underline{0.0136}$

[QUESTION 8] (5 marks)
Using the standard normal distribution N(z;0,1) table to solve for the specified values in the following probabilities:

(a) Solve for ***k*** given $P(0 < z < \boldsymbol{k}) = 0.4616$;

THE Z VALUE CORRESPONDING TO AN AREA OF 0.9616 (0.5+0.4616)

$$\boxed{K = 1.77}$$

(b) Given $\sigma = 5$, solve for $\boldsymbol{\mu}$ given $P(x > 0.75) = 0.6915$;

$$P\left(Z > \frac{0.75-\mu}{5}\right) = 0.6915$$

$$Z = -0.5 \implies -0.5 = \frac{0.75-\mu}{5}$$

$$\boxed{\mu = 3.25}$$

(c) Given $\mu = 50$, solve for $\boldsymbol{\sigma}$ in $P(x < 47.75) = 0.2266$;

$$P\left(Z < \frac{47.75-50}{\sigma}\right) = 0.2266$$

$$\frac{47.75-50}{\sigma} = -0.75$$

$$\sigma = \frac{-2.25}{-0.75}$$

$$= \boxed{3}$$