Dawson College: Linear Algebra (SCIENCE): 201-NYC-05 : Winter 2017

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Quiz 10

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

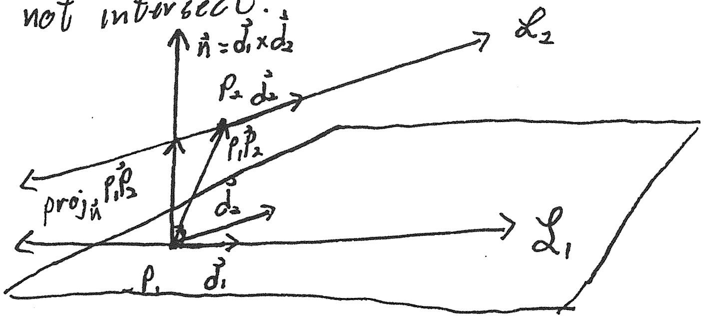
Question 1. *(5 marks)* Find the distance between the two lines: $\mathcal{L}_1: \vec{x} = \underbrace{(0,0,1)}_{P_1} + t\underbrace{(1,0,0)}_{\vec{d}_1}$ and $\mathcal{L}_2: \vec{x} = \underbrace{(0,0,3)}_{P_2} + \cancel{t}\, s\underbrace{(0,1,0)}_{\vec{d}_2}$.

$\mathcal{L}_1$ and $\mathcal{L}_2$ are not parallel since $\nexists k$ s.t. $\vec{d}_1 = k\vec{d}_2$

Do the lines intersect?

$$t = 0$$
$$0 = s$$
$$1 = 3$$

The above is inconsistent, therefore the lines do not intersect.



$\vec{P_1P_2} = P_2 - P_1 = (0,0,3) - (0,0,1) = (0,0,2)$

$\vec{n} = \vec{d}_1 \times \vec{d}_2 = \vec{i} \times \vec{j} = \vec{k} = (0,0,1)$

$$\begin{matrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{matrix}$$

$$d = \|proj_{\vec{n}} \vec{P_1P_2}\| = \left\| \frac{\vec{n}\cdot\vec{P_1P_2}}{\vec{n}\cdot\vec{n}}\, \vec{n} \right\|$$

$$= \left\| \frac{(0,0,1)\cdot(0,0,2)}{(0,0,1)\cdot(0,0,1)} (0,0,1) \right\|$$

$$= \left\| \frac{2}{1}(0,0,1) \right\|$$

$$= 2$$

Question 2. §4.1 Determine whether each set equipped with the given operations is a vector space. For those that are not vector spaces identify **one** vector space axiom that fails.

5. *(2.5 marks)* The set of all real numbers of the form (x,y), where $x \geq 0$, with the standard operations on $\mathbb{R}^2$.
7. *(2.5 marks)* The set of all triples of real numbers with the standard vector addition but with scalar multiplication defined by $k(x,y,z) = (k^2x, k^2y, k^2z)$.

#5 Let $V = \{(x,y) \mid x \geq 0 \text{ and } y \in \mathbb{R}\}$, $\vec{v} = (1,1) \in V$ and $r = -1$, then $r\cdot\vec{v} = (-1)\cdot(1,1) = (-1,-1) \notin V$ since $x = -1 < 0$. $\therefore$ not closed under scalar multiplication. $\therefore$ not a vector space.

#7 The following axiom fails: $(r+s)\vec{v} = r\vec{v} + s\vec{v}$ where $\vec{v} = (x,y,z)$ and $r,s \in \mathbb{R}$

$LHS = (r+s)\vec{v} = (r+s)(x,y,z) = ((r+s)^2x, (r+s)^2y, (r+s)^2z) = ((r^2+2rs+s^2)x, (r^2+2rs+s^2)y, (r^2+2rs+s^2)z)$

$RHS = r\vec{v} + s\vec{v} = r(x,y,z) + s(x,y,z) = (r^2x, r^2y, r^2z) + (s^2x, s^2y, s^2z) = (r^2x+s^2x, r^2y+s^2y, r^2z+s^2z)$

$LHS \neq RHS$.