Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S4: Winter 2017

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Quiz 11

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §4.2 #2d *(4 marks)* Determine whether the following are subspaces of $\mathcal{M}_{nn}$.
The set of all $n \times n$ matrices A such that $A^T = -A$

$W = \{ A \mid A \in \mathcal{M}_{n\times n} \text{ and } A^T = -A \} \subset \mathcal{M}_{n \times n}$

① Let $A \in W$, implies $A^T = -A$
$B \in W$, implies $B^T = -B$

$A+B \in \mathcal{M}_{n\times n}$

and since $(A+B)^T = A^T + B^T$
$= -A - B$ since $A, B \in W$
$= -(A+B)$

$A+B \in W$
$\therefore$ closed under addition

② Let $A \in W$, implies $A^T = -A$
$k \in \mathbb{R}$

$kA \in \mathcal{M}_{n\times n}$

and since $(kA)^T = kA^T$
$= k(-A)$ since $A \in W$
$= -kA$

$kA \in W$
$\therefore$ closed under scalar mult.

$\therefore$ by subspace test, W is a subspace.

Question 2. §4.2 #10 *(4 marks)* Express the vector $7+8x+9x^2$ as a linear combination of $\mathbf{p}_1 = 2+x+4x^2$, $\mathbf{p}_2 = 1-x+3x^2$, $\mathbf{p}_3 = 3+2x+5x^2$.

$7+8x+9x^2 = c_1p_1 + c_2p_2 + c_3p_3$

$7+8x+9x^2 = c_1(2+x+4x^2) + c_2(1-x+3x^2) + c_3(3+2x+5x^2)$

$7+8x+9x^2 = 2c_1 + c_1x + 4c_1x^2 + c_2 - c_2x + 3c_2x^2 + 3c_3 + 2c_3x + 5c_3x^2$

$7+8x+9x^2 = (2c_1 + c_2 + 3c_3) + (c_1 - c_2 + 2c_3)x + (4c_1 + 3c_2 + 5c_3)x^2$

$$\begin{bmatrix} 2 & 1 & 3 & 7 \\ 1 & -1 & 2 & 8 \\ 4 & 3 & 5 & 9 \end{bmatrix} \sim \underset{R_1 \leftrightarrow R_2}{} \begin{bmatrix} 1 & -1 & 2 & 8 \\ 2 & 1 & 3 & 7 \\ 4 & 3 & 5 & 9 \end{bmatrix} \sim \begin{matrix} \\ -2R_1+R_2 \to R_2 \\ -4R_1+R_3 \to R_3 \end{matrix} \begin{bmatrix} 1 & -1 & 2 & 8 \\ 0 & 3 & -1 & -9 \\ 0 & 7 & -3 & -23 \end{bmatrix} \sim \begin{matrix} \\ \\ 3R_3 \to R_3 \end{matrix} \begin{bmatrix} 1 & -1 & 2 & 8 \\ 0 & 3 & -1 & -9 \\ 0 & 21 & -9 & -69 \end{bmatrix}$$

$$\sim \begin{matrix} \\ \\ -7R_2+R_3 \to R_3 \end{matrix} \begin{bmatrix} 1 & -1 & 2 & 8 \\ 0 & 3 & -1 & -9 \\ 0 & 0 & -2 & -6 \end{bmatrix} \sim \begin{matrix} \\ \\ -\frac{1}{2}R_3 \to R_3 \end{matrix} \begin{bmatrix} 1 & -1 & 2 & 8 \\ 0 & 3 & -1 & -9 \\ 0 & 0 & 1 & 3 \end{bmatrix} \begin{matrix} -2R_3+R_1 \to R_1 \\ \sim R_3+R_2 \to R_2 \\ \\ \end{matrix} \begin{bmatrix} 1 & -1 & 0 & 2 \\ 0 & 3 & 0 & -6 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

$$\sim \begin{matrix} \\ \frac{1}{3}R_2 \to R_2 \\ \\ \end{matrix} \begin{bmatrix} 1 & -1 & 0 & 2 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{bmatrix} \sim \begin{matrix} R_2+R_1 \to R_1 \\ \\ \\ \end{matrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

$\therefore c_1 = 0$
$c_2 = -2$
$c_3 = 3$

Question 3. §3.1 #TF *(2 marks)* Determine whether the statement is true or false, and justify your answer.
The solution set of a consistent linear system $Ax = b$ of m equations in n unknowns is a subspace of $\mathbb{R}^n$.

False, fails the subspace test.

Let $x_1, x_2 \in W = \{ x \mid x \in \mathbb{R}^n \text{ and } Ax = b \}$

then $x_1 + x_2 \notin W$ since

$A(x_1 + x_2)$
$= Ax_1 + Ax_2$
$= b + b$
$= 2b \neq b$ if $b \neq 0$.