Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S4: Winter 2017

Name: Y. Lamontagne

Quiz 12

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §4.2 #11a *(3 marks)* Determine whether the given vectors span $\mathbb{R}^3$. $\vec{v}_1 = (2,2,2), \vec{v}_2 = (0,0,3), \vec{v}_3 = (0,1,1)$.

Let $\vec{x} = (a,b,c) \in \mathbb{R}^3$. Is $\vec{x} \in span(\{\vec{v}_1, \vec{v}_2, \vec{v}_3\})$? That is, does there $\exists\, c_1, c_2, c_3$ s.t.

$\vec{x} = c_1\vec{v}_1 + c_2\vec{v}_2 + c_3\vec{v}_3$

$(a,b,c) = c_1(2,2,2) + c_2(0,0,3) + c_3(0,1,1)$

$$\underbrace{\begin{bmatrix} 2 & 0 & 0 \\ 2 & 0 & 1 \\ 2 & 2 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}}_{c} = \underbrace{\begin{bmatrix} a \\ b \\ c \end{bmatrix}}_{d}$$

There exists c_i iff $|A| \neq 0$ by the equivalence thm

$|A| = -2 \begin{vmatrix} 2 & 0 \\ 2 & 1 \end{vmatrix} = -4 \neq 0$

$\therefore \vec{v}_1, \vec{v}_2, \vec{v}_3$ spans $\mathbb{R}^3$

Question 2. §4.3 #10 *(5 marks)* Prove: The space spanned by two vectors in $\mathbb{R}^3$ is a line through the origin, a plane through the origin, or the origin itself.

Let $V = span(\{\vec{v}_1, \vec{v}_2\})$ and $\vec{v} = c_1\vec{v}_1 + c_2\vec{v}_2 \in V \quad \forall c_1, c_2 \in \mathbb{R}$

Possible cases:

① $\vec{v}_1 = \vec{v}_2 = 0$ then $\vec{v} = \vec{0}$, the space is the origin.

② $\exists_i$ s.t. $\vec{v}_i = \vec{0}$ and $\exists_j$ s.t $\vec{v}_j \neq \vec{0}$ then $\vec{v} = c_j\vec{v}_j$, the space is a line through the origin.

③ $\vec{v}_1 = K\vec{v}_2$ then $\vec{v} = c_1\vec{v}_1 + c_2\vec{v}_2 = c_1K\vec{v}_2 + c_2\vec{v}_2 = (c_1K + c_2)\vec{v}_2$, the space is a line through the origin.

④ $\vec{v}_1 \neq K\vec{v}_2$ then $\vec{v} = c_1\vec{v}_1 + c_2\vec{v}_2$, the space is a plane that passes through the origin.

Question 3. §4.3 #TF *(2 marks)* Determine whether the statement is true or false, and justify your answer.
Every linearly dependent set contains the zero vector.

False, the set $\{(1,1),(-1,-1)\}$ is linearly dependent since $\vec{0} = c_1(1,1) + c_2(-1,-1)$ when $c_1 = c_2 = 1$