Dawson College: Linear Algebra (SCIENCE): 201-NYC-05-S4: Winter 2017

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Quiz 2

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §1.2 #TF *(2 marks)* Determine whether the statement is true or false, and justify your answer.
If an elementary row operation is applied to a matrix that is in row echelon form, the resulting matrix will still be in row echelon form.

False, $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ is in REF but after $R_1 \leftrightarrow R_2$ is performed the matrix $\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ is no longer under REF

Question 2. §1.2 #11a *(3 marks)* Determine whether the statement is true or false, and justify your answer.
If the reduced row echelon form of the augmented matrix for a linear system has a row of zeros, then the system must have infinitely many solutions.

False, the following system $\begin{aligned} x+y&=1 \\ 2x+2y&=2 \\ x+y&=2 \end{aligned}$ has augmented matrix $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 1 & 1 & 2 \end{bmatrix}$ which has REF $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ and since it has a leading 1 in the constant column there is no solution.

Question 2. §1.2 #7 *(5 marks)* Solve the given linear system by Gauss-Jordan or Gaussian elimination.

$$\begin{array}{rcrcrcrcr} x & - & y & + & 2z & - & w & = & -1 \\ 2x & + & y & - & 2z & - & 2w & = & -2 \\ -x & + & 2y & - & 4z & + & w & = & 1 \\ 3x & & & & & - & 3w & = & -3 \end{array}$$

$$\begin{array}{rcrcrcrcr} x & - & y & + & 2z & - & w & = & -1 \\ 2x & + & y & - & 2z & - & 2w & = & -2 \\ -x & + & 2y & - & 4z & + & w & = & 1 \\ 3x & & & & & - & 3w & = & -3 \end{array}$$

$$\begin{bmatrix} 1 & -1 & 2 & -1 & -1 \\ 2 & 1 & -2 & -2 & -2 \\ -1 & 2 & -4 & 1 & 1 \\ 3 & 0 & 0 & -3 & -3 \end{bmatrix}$$

$$\sim \begin{array}{r} -2R_1 + R_2 \to R_2 \\ R_1 + R_3 \to R_3 \\ -3R_1 + R_4 \to R_4 \end{array} \begin{bmatrix} 1 & -1 & 2 & -1 & -1 \\ 0 & 3 & -6 & 0 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 3 & -6 & 0 & 0 \end{bmatrix}$$

$$\sim \begin{array}{r} \frac{1}{3} R_2 \to R_2 \\ -\frac{1}{3} R_2 + R_3 \to R_3 \\ -R_2 + R_4 \to R_4 \end{array} \begin{bmatrix} 1 & -1 & 2 & -1 & -1 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\sim R_2 + R_1 \to R_1 \begin{bmatrix} 1 & 0 & 0 & -1 & -1 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$z = s \qquad s, t \in \mathbb{R}$

$w = t$

$x = -1 + t$

$y = +2s$

$\therefore (x, y, z, w) = (-1+t, 2s, s, t)$