Dawson College: Linear Algebra (SCIENCE): 201-NYC-05-S4: Winter 2017

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Quiz 4

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §1.4 #TF *(2 marks)* Determine whether the statement is true or false, and justify your answer.
The sum of two invertible matrices of the same size must be invertible.

False, Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$, both are invertible since in both case $ad - bc = 1 \neq 0$

But $A + B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ which is not invertible.

Question 2. §1.4 #TF *(2 marks)* Determine whether the statement is true or false, and justify your answer.
For all square matrices A and B of the same size it is true that $(A - B)^2 = A^2 - B^2$.

False, Let $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}$

$LHS = (A - B)^2 = \left(\begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} \right)^2 = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} = \begin{bmatrix} -4 & 0 \\ 0 & -4 \end{bmatrix}$

$RHS = A^2 - B^2 = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}^2 - \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}^2 = \begin{bmatrix} 1 & 5 \\ 0 & 9 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 5 & 9 \end{bmatrix} = \begin{bmatrix} 0 & 5 \\ -5 & 0 \end{bmatrix} \neq LHS$

Question 3. §1.4 #31 *(3 marks)* Assuming that all matrices are $n \times n$ and invertible, solve for D.

$$C^T B^{-1} A^2 B A C^{-1} D A^{-2} B^T C^{-2} = C^T$$

$\underbrace{(C^T B^{-1} A^2 B A C^{-1})^{-1} C^T B^{-1} A^2 B A C^{-1}}_{I} \underbrace{D}_{D} \underbrace{A^{-2} B^T C^{-2} (A^{-2} B^T C^{-2})^{-1}}_{I} = (C^T B^{-1} A^2 B A C^{-1})^{-1} C^T (A^{-2} B^T C^{-2}$

$= (C^{-1})^{-1} A^{-1} B^{-1} (A^2)^{-1} (B^{-1})^{-1} \overbrace{(C^T)^{-1} C^T}^{I}$

$(C^{-2})^{-1} (B^T)^{-1} ($

$$D = C A^{-1} B^{-1} A^{-2} B C^2 (B^T)^{-1} A^2$$

Question 4. §1.4 #54b *(3 marks)* A square matrix A is said to be *idempotent* if $A^2 = A$. Show that if A is idempotent, then $2A - I$ is invertible and is its own inverse.

Premise: A is idempotent, $A^2 = A$

Conclusion: 1) $2A - I$ is idempotent

2) $(2A - I)^{-1} = (2A - I)$

Need to show $(2A - I)(2A - I) = I$

$LHS = (2A - I)(2A - I)$

$= 2A2A - 2AI - I2A + II$

$= 4A^2 - 4A + I$

$= 4A - 4A + I$

$= I = RHS.$