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Quiz 5

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §1.6 #22 *(5 marks)* Let $A\mathbf{x} = \mathbf{0}$ be a homogeneous system of n linear equations in n unknowns, and let Q be an invertible $n \times n$ matrix. Show that $A\mathbf{x} = \mathbf{0}$ has just the trivial solution if and only if $(QA)\mathbf{x} = \mathbf{0}$ has just the trivial solution.

[$\Rightarrow$]

Premise: • Q is invertible
• $Ax=0$ has only the trivial solution

Conclusion:
$QAx=0$ has only the trivial solution

A is invertible since $Ax=0$ has only the trivial solutions. It follows that QA is invertible since Q is invertible as well. Hence by the equivalence theorem $QAx=0$ has only the trivial solution.

[$\Leftarrow$]

Premise: • Q is invertible
• $QAx=0$ has only the trivial solution

Conclusion:
$Ax=0$ has only the trivial solution

$Ax=0$ has only the trivial solution if and only if A is invertible. QA is invertible since $QAx=0$ has only the trivial solution.

So $(QA)^{-1}QA = I$

and it follows that A is invertible and $(QA)^{-1}Q$ is its inverse.

Question 2. §1.7 #TF *(5 marks)* Find all values of the unknown constant(s) in order for A to be symmetric.

$$A = \begin{bmatrix} 2 & a-2b+2c & 2a+b+c \\ 3 & 5 & a+c \\ 0 & -2 & 7 \end{bmatrix}$$

For A to be symmetric: $3 = a-2b+2c$
$0 = 2a+b+c$
$-2 = a+c$

Lets solve for a, b, c.

$$\begin{bmatrix} 1 & 0 & 1 & -2 \\ 1 & -2 & 2 & 3 \\ 2 & 1 & 1 & 0 \end{bmatrix} \sim \begin{matrix} \\ -R_1+R_2 \to R_2 \\ -2R_1+R_3 \to R_3 \end{matrix} \begin{bmatrix} 1 & 0 & 1 & -2 \\ 0 & -2 & 1 & 5 \\ 0 & 1 & -1 & 4 \end{bmatrix} \sim R_2 \leftrightarrow R_3 \begin{bmatrix} 1 & 0 & 1 & -2 \\ 0 & 1 & -1 & 4 \\ 0 & -2 & 1 & 5 \end{bmatrix}$$

$$\sim \begin{matrix} \\ \\ 2R_2+R_3 \to R_3 \end{matrix} \begin{bmatrix} 1 & 0 & 1 & -2 \\ 0 & 1 & -1 & 4 \\ 0 & 0 & -1 & 13 \end{bmatrix} \sim \begin{matrix} R_3+R_1 \to R_1 \\ -R_3+R_2 \to R_2 \\ -R_3 \to R_3 \end{matrix} \begin{bmatrix} 1 & 0 & 0 & 11 \\ 0 & 1 & 0 & -9 \\ 0 & 0 & 1 & -13 \end{bmatrix}$$

$\therefore$ $a = 11$, $b = -9$, $c = -13$