**Dawson College**: **Linear Algebra** (SCIENCE): 201-NYC-05-S4: Winter 2017

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# Quiz 6

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

**Question 1.** §2.1 #18 *(3 marks)* Find all the values of $\lambda$ for which $\det(A) = 0$.

$$A = \begin{bmatrix} \lambda-4 & 4 & 0 \\ -1 & \lambda & 0 \\ 0 & 0 & \lambda-5 \end{bmatrix}$$

$$\begin{aligned}
\det(A) &= a_{31}C_{31} + a_{32}C_{32} + a_{33}C_{33} \\
&= 0C_{31} + 0C_{32} + a_{33}C_{33} \\
&= (\lambda-5)C_{33} \\
&= (\lambda-5)(-1)^{3+3}\begin{vmatrix} \lambda-4 & 4 \\ -1 & \lambda \end{vmatrix} \\
&= (\lambda-5)[\lambda(\lambda-4)+4] = (\lambda-5)(\lambda^2-4\lambda+4) \\
&= (\lambda-5)(\lambda-2)^2 = 0
\end{aligned}$$

$\lambda-5=0$, $\lambda=5$ $\qquad$ $\lambda-2=0$, $\lambda=2$

**Question 2.** §2.1 #TF *(2 marks)* Determine whether the statement is true or false, and justify your answer.
Two square matrices $A$ and $B$ can have the same determinant only if they are the same size.

False, $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$ but $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1$.

**Question 3.** §2.2 #30 *(5 marks)* Confirm the identitites without evaluating the determinant directly.

$$\begin{vmatrix} a_1+b_1t & a_2+b_2t & a_3+b_3t \\ a_1t+b_1 & a_2t+b_2 & a_3t+b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = (1-t^2)\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$\begin{vmatrix} a_1+b_1t & a_2+b_2t & a_3+b_3t \\ a_1t+b_1 & a_2t+b_2 & a_3t+b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$\overset{R_1+R_2\to R_2}{=} \begin{vmatrix} a_1+b_1t & a_2+b_2t & a_3+b_3t \\ a_1+b_1+a_1t+b_1t & a_2+b_2+a_2t+b_2t & a_3+b_3+a_3t+b_3t \\ C_1 & C_2 & C_3 \end{vmatrix}$$

$$= \begin{vmatrix} a_1+b_1t & a_2+b_2t & a_3+b_3t \\ (a_1+b_1)(1+t) & (a_2+b_2)(1+t) & (a_3+b_3)(1+t) \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= (1+t)\begin{vmatrix} a_1+b_1t & a_2+b_2t & a_3+b_3t \\ a_1+b_1 & a_2+b_2 & a_3+b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$\overset{-R_1+R_2\to R_2}{=} (1+t)\begin{vmatrix} a_1+b_1t & a_2+b_2t & a_3+b_3t \\ b_1-tb_1 & b_2-tb_2 & b_3-tb_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= (1+t)\begin{vmatrix} a_1+b_1t & a_2+b_2t & a_3+b_3t \\ b_1(1-t) & b_2(1-t) & b_3(1-t) \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= (1+t)(1-t)\begin{vmatrix} a_1+b_1t & a_2+b_2t & a_3+b_3t \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$\overset{-tR_2+R_1\to R_1}{=} (1-t^2)\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$