Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S5: Winter 2017

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Quiz 7

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §2.2 #24 *(3 marks)* Solve by Cramer's rule, where it applies.

$7x_1 - 2x_2 = 3$

$3x_1 + x_2 = 5$

$$\equiv \underbrace{\begin{bmatrix} 7 & -2 \\ 3 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 3 \\ 5 \end{bmatrix}}_{b}$$

$\det A = 7(1) - (-2)(3) = 13$ So we can use Cramer's Rule

$\det A_1 = \begin{vmatrix} 3 & -2 \\ 5 & 1 \end{vmatrix} = 13$

$\det A_2 = \begin{vmatrix} 7 & 3 \\ 3 & 5 \end{vmatrix} = 26$

$\therefore\ x_1 = \dfrac{\det A_1}{\det A} = \dfrac{13}{13} = 1$

$x_2 = \dfrac{\det A_2}{\det A} = \dfrac{26}{13} = 2$

Question 2. #3.4.10 *(3 marks)* Consider two 4×4 matrices A and B, with $\det(A) = -2$ and $\det(B) = 3$. Find the determinant of M, knowing that $\det(2B^T MA^{-1}B) = \det(\operatorname{adj}(A)A^2B)$.

$2^4 \det(B^T M A^{-1} B) = \det(\operatorname{adj}(A)) \det(A^2) \det B$

$2^4 \det B^T \det M \det A^{-1} \cancel{\det B} = (\det A)^{4-1} (\det A)^2 \cancel{\det B}$

$2^4 \det B \det M \dfrac{1}{\det A} = (\det A)^3 (\det A)^2$

$\det M = \dfrac{(\det A)^6}{2^4 \det B}$

$= \dfrac{(-2)^6}{2^4\, 3} = \dfrac{4}{3}$

Question 3. §3.1 #TF *(2 marks)* Determine whether the statement is true or false, and justify your answer.
The vectors (a,b) and $(a,b,0)$ are equivalent.

False, since (a,b) is a vector in 2-space and $(a,b,0)$ is a vector in 3-space.

Question 4. §3.1 #TF *(2 marks)* Determine whether the statement is true or false, and justify your answer.
The linear combinations $a_1\vec{v}_1 + a_2\vec{v}_2$ and $b_1\vec{v}_1 + b_2\vec{v}_2$ can only be equal if $a_1 = b_1$ and $a_2 = b_2$.

False, if $a_1 = 1$, $b_1 = 2$, $a_2 = b_2 = 1$ and $\vec{v}_1 = (0,0)$, $\vec{v}_2 = (1,1)$ then $a_1\vec{v}_1 + a_2\vec{v}_2 = b_1\vec{v}_1 + b_2\vec{v}_2$ but $a_1 \neq b_1$.