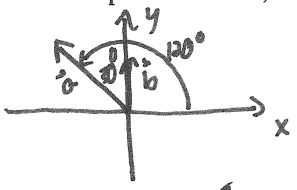
Dawson College: Linear Algebra (SCIENCE): 201-NYC-05-S4: Winter 2017

Name: Y. Lamontagne

Quiz 8

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §3.2 #15 *(3 marks)* Suppose that a vector $\vec{a}$ in the xy-plane has a length of 9 units and points in a direction that is 120° counterclockwise from the positive x-axis, and a vector $\vec{b}$ in that plane has a length of 5 units and points in the positive y-direction. Find $\vec{a}\cdot\vec{b}$.

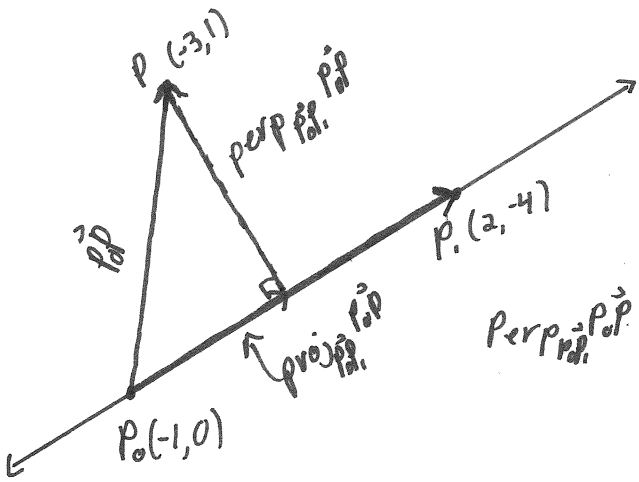


$\vec{a}\cdot\vec{b} = \|\vec{a}\|\|\vec{b}\|\cos\theta$

$\vec{a}\cdot\vec{b} = 9\cdot 5\cos 30°$

$\vec{a}\cdot\vec{b} = 45\,\frac{\sqrt{3}}{2}$

Question 2. §3.3 # 29 *(5 marks)* Using projections find the distance between the point and the line. $(-3, 1)$; $4x+3y+4=0$



Let $y=0 \Rightarrow 4x+3(0)+4=0$, $x=-1$ $\therefore P_0(-1,0)$

Let $x=2 \Rightarrow 4(2)+3y+4=0$, $y=-4$ $\therefore P_1(2,-4)$

$\vec{P_0P} = P-P_0 = (-3,1)-(-1,0) = (-2,1)$

$\vec{P_0P_1} = P_1-P_0 = (2,-4)-(-1,0) = (3,-4)$

$\text{perp}_{\vec{P_0P_1}}\vec{P_0P} = \vec{P_0P} - \text{proj}_{\vec{P_0P_1}}\vec{P_0P}$

$= (-2,1) - \frac{\vec{P_0P}\cdot\vec{P_0P_1}}{\vec{P_0P_1}\cdot\vec{P_0P_1}}\vec{P_0P_1}$

$= (-2,1) - \frac{-6-4}{9+16}(3,-4)$

$= (-2,1) + \frac{2}{5}(3,-4) = \left(-\frac{4}{5}, -\frac{3}{5}\right) = \frac{1}{5}(-4,-3)$

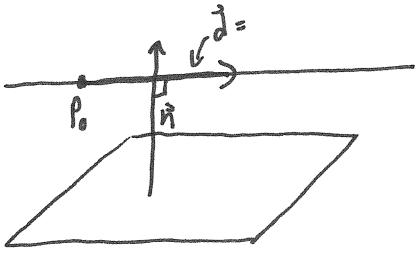
$\therefore$ distance $= \|\text{perp}_{\vec{P_0P_1}}\vec{P_0P}\|$

$= \left\|\frac{1}{5}(-4,-3)\right\|$

$= \frac{1}{5}\sqrt{(-4)^2+(-3)^2} = \frac{\sqrt{25}}{5}$

$= 1$

Question 3. # 4.5.15 *(3 marks)* Given the plane $2x_1 - x_2 + 3x_3 = 5$, and the line $\vec{x} = (3,0,4) + t(-1,1,1)$, $t \in \mathbb{R}$, determine if the line is parallel to the plane, orthogonal to the plane, or neither parallel nor orthogonal. If the answer is "neither", determine the angle between the direction vector of the line and the normal vector of the plane.



The line and the plane are parallel if the normal of the plane and direction vector are perpendicular.

$\vec{n}\cdot\vec{d} = (2,-1,3)\cdot(-1,1,1) = 0$

$\therefore$ line and plane are parallel.