Dawson College: Linear Algebra (SCIENCE): 201-NYC-05-S4: Winter 2017

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Quiz 9

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §3.4 #21

a. *(3 marks)* The equation $x+y+z=1$ can be viewed as a linear system of one equation in three unknowns. Express a general solution of this equation as a particular solution plus a general solution of the associated homogeneous system.

b. *(2 marks)* Give a geometric interpretation of the result in part a.

a) A particular solution of $Ax=b$ is $(x,y,z)=(1,0,0)$ since it satisfies the equation
$LHS = 1+0+0 = 1 = RHS$

The general solution of the homogeneous system $Ax=0$ is $x+y+z=0$

Let $y=s$, $z=t$, $s,t \in \mathbb{R}$

$x+s+t=0$

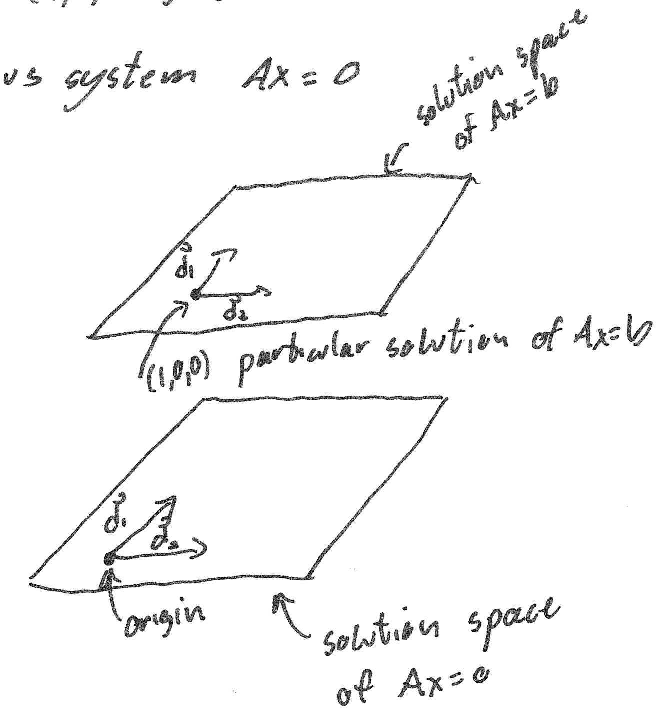
$x=-s-t$

$\therefore (x,y,z) = s\underbrace{(-1,1,0)}_{\vec{d}_1} + t\underbrace{(0,-1,1)}_{\vec{d}_2}$

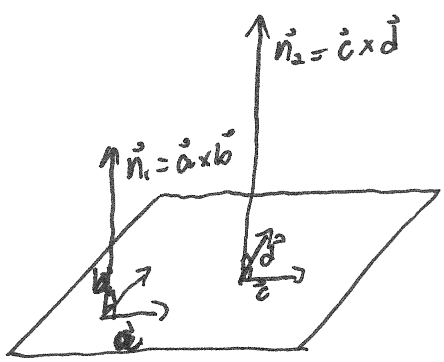
$\therefore$ the solution of $Ax=b$ is

$$(x,y,z) = \underbrace{(1,0,0)}_{\substack{\text{particular}\\ \text{solution of}\\ Ax=b}} + \underbrace{s(-1,1,0)+t(0,-1,1)}_{\substack{\text{general solution}\\ \text{of } Ax=0}}$$

b)



Question 2. §3.5 # 29 *(5 marks)* Prove: If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ lie in the same plane, then $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d}) = \vec{0}$. (*)



since $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ lie in the same plane

① $\vec{a}\times\vec{b} = \vec{0}$ or $\vec{n}_1$ a normal to the plane ($\vec{0}$ if colinear)

② $\vec{c}\times\vec{d} = \vec{0}$ or $\vec{n}_2$ a normal to the plane ($\vec{0}$ if colinear)

If any of the two are equal to $\vec{0}$ then (*) is true.

If the two (①, ②) are not equal to $\vec{0}$ then $\vec{n}_1 \parallel \vec{n}_2$ since they are both normals of the same plane. $\therefore \vec{n}_1 = k\vec{n}_2$

Hence $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d}) = \vec{n}_1 \times \vec{n}_2$

$= (k\vec{n}_2)\times\vec{n}_2$

$= k(\vec{n}_2\times\vec{n}_2) = k(\vec{0}) = \vec{0}$.