Name: _____

Test 2

This test is graded out of 43 marks. No books, notes, watches or cell phones are allowed. You are only permitted to use the Sharp EL-531XG or Sharp EL-531X calculator. Give the work in full; – unless otherwise stated, reduce each answer to its simplest, exact form; – and write and arrange your exercise in a legible and orderly manner. If you need more space for your answer use the back of the page.

Question 1. Given

$$A = \begin{bmatrix} 2 & 1 & 4 & 4 \\ -3 & 0 & -3 & -4 \\ 3 & 2 & 0 & 0 \\ 4 & 0 & 5 & 0 \end{bmatrix}.$$

a. *(5 marks)* Evaluate $\det(A)$.

b. *(5 marks)* If M is a 4×4 matrix such that $\det(M) = 3$ then evaluate $\det(\det(5A^T)\text{adj}(MA^{-1}))$. Justify!

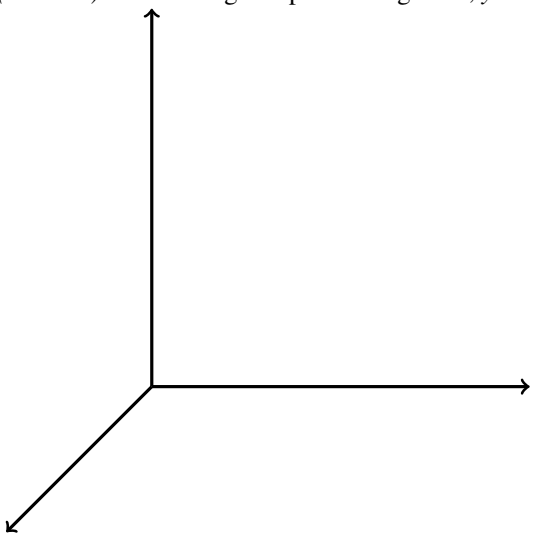
Question 2. *(3 marks)* Prove: There does not exist $n \times n$ invertible matrices A and B where A is symmetric, B is skew symmetric, n is odd such that AB is symmetric.

Question 3. *(3 marks)* Prove or disprove: If $(AB)\mathbf{x} = \mathbf{0}$ has only the trivial solution then $A\mathbf{x} = \mathbf{0}$ and $B\mathbf{x} = \mathbf{0}$ have only the trivial solution.

Question 4. *(3 marks)* Prove or disprove: There does not exist two unit vectors $\vec{u}$, $\vec{v} \in \mathbb{R}^n$ such that $\vec{u} \cdot \vec{v} = -2$.

Question 5. Given the plane $4x+3y+2z=12$

a. *(2 marks)* Sketch the given plane using the x, y and z-intercepts. Label the axes!



b. *(5 marks)* Using projections find the shortest distance between the y-intercept and the line which passes through the x and z-intercepts.

c. *(2 marks)* Find the equation of the line which passes through the y-intercept and the closest point to the y-intercept from the line which passes through the x and z-intercepts.

Question 6. *(4 marks)* Using elementary operations show that

$$-sr\begin{vmatrix} a & b \\ c & d \end{vmatrix} = \begin{vmatrix} sb+2d & rsa+2rc \\ d & rc \end{vmatrix}$$

Question 7. *(3 marks)* Prove: If $\vec{u}, \vec{v} \in \mathbb{R}^n$ such that $||\vec{u}|| = \sqrt{2}$ and the angle between $\vec{u}$ and $\vec{v}$ is $\frac{\pi}{4}$ then $||\vec{u}+\vec{v}||^2 = ||\vec{v}||^2 + 2||\vec{v}|| + 2$.

Question 8. Given the plane $x+y+z=0$ and the line $(x,\ y,\ z)=(1+t,2+2t,3+3t)$ where $t\in\mathbb{R}$.

a. *(2 marks)* Determine whether the line is perpendicular to the plane, parallel or neither. Justify!

b. *(3 marks)* Find the point of intersection between the line and the plane if it exists.

c. *(3 marks)* Find the smallest angle between the line and the plane.

Bonus Question. *(5 marks)*
Prove: If A and B are two matrices such that $A+B=AB$ then A and B commute.