Name: ____________________

Test 3

This test is graded out of 38 marks. No books, notes, watches or cell phones are allowed. You are only permitted to use the Sharp EL-531XG or Sharp EL-531X calculator. Give the work in full; – unless otherwise stated, reduce each answer to its simplest, exact form; – and write and arrange your exercise in a legible and orderly manner. If you need more space for your answer use the back of the page.

Question 1. Given:

$$\begin{array}{llll} \mathscr{L}_1 & : & (x,y,z) = (1,0,1) & + \quad t_1(-2,-1,0) \\ \mathscr{L}_2 & : & (x,y,z) = (-2,-1,2) & + \quad t_2(1,0,1) \text{ where } t_1,\ t_2 \in \mathbb{R}. \end{array}$$

a. *(3 marks)* Determine whether the two lines intersect, are parallel or are skew lines.

b. *(5 marks)* Find a point on each line which is closest to the other line, are those points unique?

c. *(3 marks)* Find the equation of a plane that contains the line which passes through the point on each line which is closest to the other line. Is the plane unique?

Question 2. *(3 marks)* Given a parallelepiped generated by the vectors $\vec{u} = (1,-1,-1), \vec{v}$ and $\vec{w} \in \mathbb{R}^3$ with volume 2017 and $\frac{\vec{v}\times\vec{w}}{||\vec{v}\times\vec{w}||} = \frac{1}{\sqrt{14}}(1,2,3)$. Find the area of the base defined by $\vec{v}$ and $\vec{w}$.

Question 3. Given two planes:

$$\begin{array}{lcll} \mathscr{P}_1 & : & x+z & =1 \\ \mathscr{P}_2 & : & y+z & =1 \end{array}$$

a. *(1 mark)* Give a point of intersection of the two planes, by inspection.

b. *(1 mark)* Give a geometrical argument to explain why the intersection of the two planes is a line.

c. *(2 marks)* Find the direction vector for the intersection of the two planes without solving for the solution set. Justify.

d. *(1 mark)* Find the solution set of the system of linear equations determined by $\mathscr{P}_1$ and $\mathscr{P}_2$ by only using part a) and part c). Justify.

Question 4. The number of leading 1's in a row echelon form of A is called the *rank* of A. Let $V = \{M \mid M \in \mathscr{M}_{2\times 2} \text{ and } \operatorname{rank}(M) = 2\}$ with the following vector addition and scalar multiplication:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae & bf \\ cg & dh \end{bmatrix} \text{ and } k \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a^k & b^k \\ c^k & d^k \end{bmatrix}$$

a. *(3 marks)* Is the zero vector an element of V? Justify.

b. *(3 marks)* Determine whether the following axiom holds: $(rs)M = r(sM)$ where $r,\ s \in \mathbb{R}$ and $M \in V$

Question 5. *(3 marks)* Determine whether $W = \left\{a_0 + a_1x + a_2x^2 \mid a_0a_1a_2 = 0\right\}$ is a subspace of P_2.

Question 6. Given $\vec{u} = (1, -1, 1), \vec{v} = (-1, 1, \lambda), \vec{w} = (1, 2, 3) \in \mathbb{R}^3$ and the set $S = \{\vec{u},\ \vec{v},\ (\vec{u} \cdot \vec{v})\vec{w}\}$.

a. *(2 marks)* Find the value(s), if any, of λ for which S spans $\mathbb{R}^3$

b. *(2 marks)* Find the value(s), if any, of λ for which S spans a plane.

c. *(2 marks)* Find the value(s), if any, of λ for which S spans a line.

Question 7. Given $W = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \mid a + 2b + 3c + 4d = 0 \text{ and } b + c + d = 0 \right\}$ a subspace of $\mathscr{M}_{2\times 2}$.

a. *(3 marks)* Find a basis B for W.

b. *(1 mark)* State the $\dim(W)$ and $\dim(\mathscr{M}_{2\times 2})$.

Bonus Question. *from Wikipedia (3 marks)*
Given $R = \{x \mid x \notin x\}$. Discuss the following statement: $R \in R \longleftrightarrow R \notin R$.