Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S5: Winter 2017

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Quiz 10

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

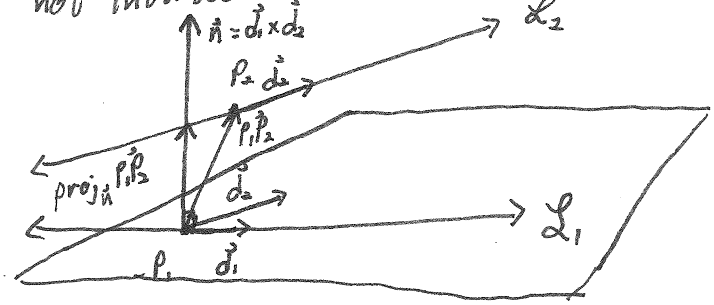
Question 1. *(5 marks)* Find the distance between the two lines: $\mathcal{L}_1: \vec{x} = \underbrace{(0,0,1)}_{P_1} + t\underbrace{(1,0,0)}_{\vec{d}_1}$ and $\mathcal{L}_2: \vec{x} = \underbrace{(0,0,3)}_{P_2} + \overset{s}{t}\underbrace{(0,1,0)}_{\vec{d}_2}$.

$\mathcal{L}_1$ and $\mathcal{L}_2$ are not parallel since $\nexists k$ s.t. $\vec{d}_1 = k\vec{d}_2$

Do the lines intersect?

$t = 0$
$0 = s$
$1 = 3$

The above is inconsistent, therefore the lines do not intersect.



$\vec{P_1P_2} = P_2 - P_1 = (0,0,3) - (0,0,1) = (0,0,2)$

$\vec{n} = \vec{d}_1 \times \vec{d}_2 = \vec{i} \times \vec{j} = \vec{k} = (0,0,1)$

$\begin{matrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{matrix}$

$$d = \|proj_{\vec{n}} \vec{P_1P_2}\| = \left\| \frac{\vec{n}\cdot\vec{P_1P_2}}{\vec{n}\cdot\vec{n}} \vec{n} \right\|$$

$$= \left\| \frac{(0,0,1)\cdot(0,0,2)}{(0,0,1)\cdot(0,0,1)} (0,0,1) \right\|$$

$$= \left\| \frac{2}{1}(0,0,1) \right\|$$

$$= 2$$

Question 2. §4.1 #2e *(5 marks)* Let V be the set of all ordered pairs of real numbers, and consider the following addition and scalar multiplication operations on $\vec{u} = (u_1, u_2)$ and $\vec{v} = (v_1, v_2)$:

$$\vec{u} + \vec{v} = (u_1 + v_1 + 1, u_2 + v_2 + 1) \text{ and } k\vec{u} = (ku_1, ku_2)$$

Find two vector space axioms that fail to hold.

The following axiom does not hold: $(r+s)\vec{v} = r\vec{v} + s\vec{v}$

$LHS = (r+s)\vec{v}$
$= ((r+s)v_1, (r+s)v_2)$
$= (rv_1 + sv_1, rv_2 + sv_2)$

$RHS = r\vec{v} + s\vec{v}$
$= (rv_1, rv_2) + (sv_1, sv_2)$
$= (rv_1 + sv_1 + 1, rv_2 + sv_2 + 1)$

$LHS \neq RHS$

The following axiom does not hold: $r(\vec{u}+\vec{v}) = r\vec{u} + r\vec{v}$

$LHS = r(\vec{u}+\vec{v})$
$= r((u_1, u_2) + (v_1, v_2))$
$= r(u_1 + v_1 + 1, u_2 + v_2 + 1)$
$= (r(u_1 + v_1 + 1), r(u_2 + v_2 + 1))$
$= (ru_1 + rv_1 + r, ru_2 + rv_2 + r)$

$RHS = r\vec{u} + r\vec{v}$
$= r(u_1, u_2) + r(v_1, v_2)$
$= (ru_1, ru_2) + (rv_1, rv_2)$
$= (ru_1 + rv_1 + 1, ru_2 + rv_2 + 1)$

$LHS \neq RHS$.