Dawson College: Linear Algebra (SCIENCE): 201-NYC-05-S5: Winter 2017

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Quiz 11

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §4.2 #2d *(4 marks)* Determine whether the following are subspaces of $\mathscr{M}_{nn}$.
The set of all symmetric $n \times n$ matrices.

$W = \{A \mid A \in M_{n\times n} \text{ and } A^T = A\} \subset M_{n\times n}$

① Let $A \in W$, implies $A^T = A$
$B \in W$, implies $B^T = B$

$A + B \in M_{n\times n}$

and since $(A+B)^T = A^T + B^T$
$= A + B$ since $A, B \in W$

$\therefore A + B \in W$
$\therefore$ closed under addition

② Let $A \in W$, implies $A^T = A$
$k \in \mathbb{R}$

$kA \in M_{n\times n}$

and since $(kA)^T = kA^T$
$= kA$ since $A \in W$

$\therefore kA \in W$
$\therefore$ closed under scalar mult.

Question 2. §4.2 #10 *(4 marks)* Express the vector $6+11x+6x^2$ as a linear combination of $\mathbf{p}_1 = 2+x+4x^2$, $\mathbf{p}_2 = 1-x+3x^2$, $\mathbf{p}_3 = 3+2x+5x^2$.

$6 + 11x + 6x^2 = c_1 p_1 + c_2 p_2 + c_3 p_3$

$6 + 11x + 6x^2 = c_1(2 + x + 4x^2) + c_2(1 - x + 3x^2) + c_3(3 + 2x + 5x^2)$

$6 + 11x + 6x^2 = 2c_1 + c_1 x + 4c_1 x^2 + c_2 - c_2 + 3c_2 x^2 + 3c_3 + 2c_3 x + 5c_3 x^2$

$6 + 11x + 6x^2 = (2c_1 + c_2 + 3c_3) + (c_1 - c_2 + 2c_3)x + (4c_1 + 3c_2 + 5c_3)x^2$

$$\begin{bmatrix} 2 & 1 & 3 & 6 \\ 1 & -1 & 2 & 11 \\ 4 & 3 & 5 & 6 \end{bmatrix} \sim_{R_1 \leftrightarrow R_2} \begin{bmatrix} 1 & -1 & 2 & 11 \\ 2 & 1 & 3 & 6 \\ 4 & 3 & 5 & 6 \end{bmatrix} \sim_{\substack{-2R_1 + R_2 \to R_2 \\ -4R_1 + R_3 \to R_3}} \begin{bmatrix} 1 & -1 & 2 & 11 \\ 0 & 3 & -1 & -16 \\ 0 & 7 & -3 & -38 \end{bmatrix} \sim_{3R_3 \to R_3} \begin{bmatrix} 1 & -1 & 2 & 11 \\ 0 & 3 & -1 & -16 \\ 0 & 21 & -9 & -114 \end{bmatrix}$$

$$\sim_{-7R_2 + R_3 \to R_3} \begin{bmatrix} 1 & -1 & 2 & 11 \\ 0 & 3 & -1 & -16 \\ 0 & 0 & -2 & -2 \end{bmatrix} \sim_{-\frac{1}{2}R_3 \to R_3} \begin{bmatrix} 1 & -1 & 2 & 11 \\ 0 & 3 & -1 & -16 \\ 0 & 0 & 1 & 1 \end{bmatrix} \sim_{\substack{-2R_3 + R_1 \to R_1 \\ R_3 + R_2 \to R_2}} \begin{bmatrix} 1 & -1 & 0 & 9 \\ 0 & 3 & 0 & -15 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$\sim_{\frac{1}{3}R_2 \to R_2} \begin{bmatrix} 1 & -1 & 0 & 9 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 1 \end{bmatrix} \sim_{R_2 + R_1 \to R_1} \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$\therefore c_1 = 4,\ c_2 = -5,\ c_3 = 1$

Question 3. §3.1 #TF *(2 marks)* Determine whether the statement is true or false, and justify your answer.
Every subset of a vector space V that contains the zero vector in V is a subspace of V.

False, Let $V = \mathbb{R}$ (a vector space) and $W = \mathbb{R}^+ = \{x \mid x \geq 0\}$ a subset of V that contains the zero vector. W is not a subspace since if we let $r = -1 \in \mathbb{R}$ and $\vec{u} = 1 \in W$ then $r \cdot \vec{u} = -1 \cdot 1 \notin W$. So not closed under scalar multiplication.