Dawson College: Linear Algebra (SCIENCE): 201-NYC-05-S5: Winter 2017

Name: Y. Lamontagne

Quiz 12

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §4.2 #11b *(3 marks)* Determine whether the given vectors span $\mathbb{R}^3$. $\vec{v}_1 = (2,-1,3), \vec{v}_2 = (4,1,2), \vec{v}_3 = (8,-1,8)$.

Let $\vec{x} = (a,b,c) \in \mathbb{R}^3$. Is $\vec{x} \in \text{span}(\{\vec{v}_1, \vec{v}_2, \vec{v}_3\})$? That is, does there $\exists c_1, c_2, c_3$ s.t.

$\vec{x} = c_1\vec{v}_1 + c_2\vec{v}_2 + c_3\vec{v}_3$

$(a,b,c) = c_1(2,-1,3) + c_2(4,1,2) + c_3(8,-1,8)$

$$\underbrace{\begin{bmatrix} 2 & 4 & 8 \\ -1 & 1 & -1 \\ 3 & 2 & 8 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}}_{c} = \underbrace{\begin{bmatrix} a \\ b \\ c \end{bmatrix}}_{d}$$

There exists c_i iff $|A| \neq 0$ by the equivalence thm.

$|A| = (-1)(-1)\begin{vmatrix} 4 & 8 \\ 2 & 8 \end{vmatrix} + \begin{vmatrix} 2 & 8 \\ 3 & 8 \end{vmatrix} + (-1)(-1)\begin{vmatrix} 2 & 4 \\ 3 & 2 \end{vmatrix} = 16 - 8 - 8 = 0$

$\therefore \mathbb{R}^3$ is not spanned by $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$

Question 2. §4.3 #10 *(5 marks)* Show that if $\{\vec{v}_1, \vec{v}_2\}$ is linearly independent and $\vec{v}_3$ does not lie span($\{\vec{v}_1, \vec{v}_2\}$), then $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is linearly independent.

Suppose that $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is linearly dependent then $\exists c_i \neq 0$ such that

$\vec{0} = c_1\vec{v}_1 + c_2\vec{v}_2 + c_3\vec{v}_3$

If $c_1 \neq 0$ or $c_2 \neq 0$ and $c_3 = 0$ then $\vec{0} = c_1\vec{v}_1 + c_2\vec{v}_2$ has a nontrivial combination $\lightning$ since $\{\vec{v}_1, \vec{v}_2\}$ is linearly independent.

If $c_3 \neq 0$ then $\vec{0} = c_1\vec{v}_1 + c_2\vec{v}_2 + c_3\vec{v}_3$

$-c_3\vec{v}_3 = c_1\vec{v}_1 + c_2\vec{v}_2$

$\vec{v}_3 = \frac{c_1}{-c_3}\vec{v}_1 + \frac{c_2}{-c_3}\vec{v}_2$ which implies $\vec{v}_3 \in \text{span}(\{\vec{v}_1, \vec{v}_2\})$ $\lightning$

since $\vec{v}_3$ does not lie in the span of $\{\vec{v}_1, \vec{v}_2\}$.

$\therefore \{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is linearly independent.

Question 3. §4.3 #TF *(2 marks)* Determine whether the statement is true or false, and justify your answer.
A set containing a single vector is linearly independent.

False, $\{\vec{0}\}$ is linearly dependent since $\vec{0} = c_1\vec{0}$ where $c_1 = 1$.