Dawson College: Linear Algebra (SCIENCE): 201-NYC-05-S5: Winter 2017

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Quiz 2

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §1.2 #TF *(2 marks)* Determine whether the statement is true or false, and justify your answer.
If an elementary row operation is applied to a matrix that is in row echelon form, the resulting matrix will still be in row echelon form.

False, The following matrix $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ is in REF but after $2R_2 \to R_2$, the matrix $\begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$ is no longer in REF.

Question 2. §1.2 #TF *(3 marks)* Determine whether the statement is true or false, and justify your answer.
Every matrix has a unique row echelon form.

False,

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \sim_{R_1 \leftrightarrow R_2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \sim_{R_1 + R_2 \to R_2} \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \sim_{R_1 \leftrightarrow R_2} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

Same matrix but two different REF, $\therefore$ REF is not unique.

Question 3. §1.2 #20 *(5 marks)* Solve the given linear system by any method.

$$\begin{array}{rcrcrcrcl} & & v & + & 3w & - & 2x & = & 0 \\ 2u & + & v & - & 4w & + & 3x & = & 0 \\ 2u & + & 3v & + & 2w & - & x & = & 0 \\ -4u & - & 3v & + & 5w & - & 4x & = & 0 \end{array}$$

$$\begin{array}{rcrcrcrcl} & & v & + & 3w & - & 2x & = & 0 \\ 2u & + & v & - & 4w & + & 3x & = & 0 \\ 2u & + & 3v & + & 2w & - & x & = & 0 \\ -4u & - & 3v & + & 5w & - & 4x & = & 0 \end{array}$$

$$\begin{bmatrix} 0 & 1 & 3 & -2 & 0 \\ 2 & 1 & -4 & 3 & 0 \\ 2 & 3 & 2 & -1 & 0 \\ -4 & -3 & 5 & -4 & 0 \end{bmatrix} \sim \begin{array}{c} R_1 \leftrightarrow R_2 \\ \\ \\ \\ \end{array} \begin{bmatrix} 2 & 1 & -4 & 3 & 0 \\ 0 & 1 & 3 & -2 & 0 \\ 2 & 3 & 2 & -1 & 0 \\ -4 & -3 & 5 & -4 & 0 \end{bmatrix}$$

$$\sim \begin{array}{r} \\ \\ -R_1+R_3 \rightarrow R_3 \\ 2R_1+R_4 \rightarrow R_4 \end{array} \begin{bmatrix} 2 & 1 & -4 & 3 & 0 \\ 0 & 1 & 3 & -2 & 0 \\ 0 & 2 & 6 & -4 & 0 \\ 0 & -1 & -3 & 2 & 0 \end{bmatrix} \sim \begin{array}{r} \\ \\ -2R_2+R_3 \rightarrow R_3 \\ R_2+R_4 \rightarrow R_4 \end{array} \begin{bmatrix} 2 & 1 & -4 & 3 & 0 \\ 0 & 1 & 3 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\sim \begin{array}{r} -R_2+R_1 \rightarrow R_1 \\ \\ \\ \\ \end{array} \begin{bmatrix} 2 & 0 & -7 & 5 & 0 \\ 0 & 1 & 3 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{array}{r} \frac{1}{2}R_1 \rightarrow R_1 \\ \\ \\ \\ \end{array} \begin{bmatrix} 1 & 0 & -7/2 & 5/2 & 0 \\ 0 & 1 & 3 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Let $x_3 = s$ $\quad s,t \in \mathbb{R}$
$x_4 = t$

From augmented matrix

$$\begin{cases} x_1 \quad - \frac{7}{2}x_3 + \frac{5}{2}x_4 = 0 \\ x_2 + 3x_3 - 2x_4 = 0 \end{cases}$$

$$\begin{aligned} x_1 &= \tfrac{7}{2}s - \tfrac{5}{2}t \\ x_2 &= -3s + 2t \\ x_3 &= s \\ x_4 &= t \end{aligned} \qquad s,t \in \mathbb{R}$$