

## Quiz 3

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

**Question 1.** §1.3 #TF (3 marks) Determine whether the statement is true or false, and justify your answer.

If  $B$  has a column of zeros, then so does  $AB$  if this product is defined. *True,*

Suppose the  $j^{\text{th}}$  column of  $B$  is zeros then the  $j^{\text{th}}$  column of the product  $AB$  is  $A[\text{j}^{\text{th}} \text{ column of } B] = A \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}$   $\therefore$  the  $j^{\text{th}}$  column of  $AB$  has a row of zeros.

**Question 2.** §1.3 #TF (2 marks) Determine whether the statement is true or false, and justify your answer.

If  $A$ ,  $B$ , and  $C$  are matrices of the same order such that  $A - C = B - C$ , then  $A = B$ .

*True,*  $A - C = B - C$

$$[a_{ij}] - [c_{ij}] = [b_{ij}] - [c_{ij}]$$

$$[a_{ij} - c_{ij}] = [b_{ij} - c_{ij}]$$

*equal iff  $a_{ij} - c_{ij} = b_{ij} - c_{ij} \forall i, j$*

$$a_{ij} = b_{ij}$$

$$\therefore A = B$$

**Question 3.** §1.3 #TF (3 marks) Determine whether the statement is true or false, and justify your answer.

If  $A$  and  $B$  are square matrices of the same order, then  $(AB)^T = A^T B^T$ .

*False,* Let  $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ ,  $B = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$

$$AB = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad (AB)^T = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, B^T = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$$

$$A^T B^T = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

**Question 4.** §1.3 #5e (2 marks) Consider the matrices

$$A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix}, C = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix}, D = \begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix}, E = \begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix}$$

*$\therefore$  in general  $(AB)^T \neq A^T B^T$*

Compute the given expression (if possible).

$$A(BC)$$

$$= A \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix}$$

$$= A \begin{bmatrix} 1 & 15 & 3 \\ 6 & 2 & 10 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 15 & 3 \\ 6 & 2 & 10 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 45 & 9 \\ 11 & -11 & 17 \\ 7 & 17 & 13 \end{bmatrix}$$