Dawson College: Linear Algebra (SCIENCE): 201-NYC-05-S5: Winter 2017

Name: Y. Lamontagne

Quiz 4

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §1.4 #TF *(3 marks)* Determine whether the statement is true or false, and justify your answer.
If A and B are invertible matrices of the same size, then AB is invertible and $(AB)^{-1} = A^{-1}B^{-1}$.

False, Let $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$ then $AB = \begin{bmatrix} 7 & 2 \\ 3 & 1 \end{bmatrix}$ and $(AB)^{-1} = \begin{bmatrix} 1 & -2 \\ -3 & 7 \end{bmatrix}$

but $A^{-1} = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}$, $A^{-1}B^{-1} = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}$

$= \begin{bmatrix} 7 & -2 \\ -3 & 1 \end{bmatrix} \neq (AB)^{-1}$

Question 2. §1.4 #TF *(3 marks)* Determine whether the statement is true or false, and justify your answer.
A square matrix containing a row or column of zeros cannot be invertible.

True,
Suppose A has a row of zeros (the i^{th} row) then for any matrix B, the i^{th} row of AB = [i^{th} row of A]B = $[0\ 0 \cdots 0]B = [0\ 0 \cdots 0]$. Hence impossible to obtain the identity.
Suppose A has a column of zeros (the j^{th} column) then for any matrix B, the j^{th} column of $BA = B$ [j^{th} column of A] $= B\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$. Hence impossible to obtain the identity.

Question 3. §1.4 #TF *(2 marks)* Determine whether the statement is true or false, and justify your answer.
For all square matrices A and B of the same size it is true that $(A+B)^2 = A^2 + 2AB + B^2$.

False
$LHS = (A+B)^2 = (A+B)(A+B) = A^2 + AB + BA + B^2$. Since AB is not necessarily equal to BA, it follows that LHS is not necessarily equal to the RHS.
Let $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$ then $AB = \begin{bmatrix} 7 & 2 \\ 3 & 1 \end{bmatrix}$ but $BA = \begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}$

Question 4. §1.4 #33 *(2 marks)* Simplify:

$$(AB)^{-1}(AC^{-1})(D^{-1}C^{-1})^{-1}D^{-1} = B^{-1}\underbrace{A^{-1}A}C^{-1}(C^{-1})^{-1}(D^{-1})^{-1}D^{-1}$$
$$= B^{-1}\ I\ \underbrace{C^{-1}C}\ \underbrace{DD^{-1}}$$
$$= B^{-1} \cdot I \cdot I$$
$$= B^{-1}$$