

Quiz 5

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §1.6 #19 (5 marks) Solve the given matrix equation for X .

$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & 3 & 0 \\ 0 & 2 & -1 \end{bmatrix} X = \begin{bmatrix} 2 & -1 & 5 & 7 & 8 \\ 4 & 0 & -3 & 0 & 1 \\ 3 & 5 & -7 & 2 & 1 \end{bmatrix}$$

Question 2. §1.7 #TF1 (2 marks) Determine whether the statement is true or false, and justify your answer.
If A^2 is a symmetric matrix, then A is a symmetric matrix.

Question 3. §1.6 #33 (3 marks) Prove: If $A^T A = A$, then A is symmetric and $A = A^2$.

Question 3. (5 marks)¹ (5 marks) Let

$$A = \begin{bmatrix} 1 & 1 & a \\ 1 & a & a \\ a & a & a \\ a & a & a^2 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ a^2 - 2a \end{bmatrix}$$

- For what value(s) of a does the system $A\mathbf{x} = \mathbf{b}$ have no solution?
- For what value(s) of a does the system $A\mathbf{x} = \mathbf{b}$ have a unique solution?
- For what value(s) of a does the system $A\mathbf{x} = \mathbf{b}$ have infinitely many solutions?

¹From a John Abbot Final Examination