Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S5: Winter 2017

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Quiz 5

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §1.6 #19 *(5 marks)* Solve the given matrix equation for X.

$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & 3 & 0 \\ 0 & 2 & -1 \end{bmatrix} X = \overbrace{\begin{bmatrix} 2 & -1 & 5 & 7 & 8 \\ 4 & 0 & -3 & 0 & 1 \\ 3 & 5 & -7 & 2 & 1 \end{bmatrix}}^{B}$$

Let $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 3 & 0 \\ 0 & 2 & -1 \end{bmatrix}$

$[A|I]$

$$= \left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 2 & 3 & 0 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\sim \sim -2R_1+R_2 \to R_2 \left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 5 & -2 & -2 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\sim 5R_3 \to R_3 \left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 5 & -2 & -2 & 1 & 0 \\ 0 & 10 & -5 & 0 & 0 & 5 \end{array}\right]$$

$$\sim -2R_2+R_3 \to R_3 \left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 5 & -2 & -2 & 1 & 0 \\ 0 & 0 & -1 & 4 & -2 & 5 \end{array}\right]$$

$$\begin{matrix} R_3+R_1 \to R_1 \\ \sim -2R_3+R_2 \to R_2 \\ -R_3 \to R_3 \end{matrix} \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 5 & -2 & 5 \\ 0 & 5 & 0 & -10 & 5 & -10 \\ 0 & 0 & 1 & -4 & 2 & -5 \end{array}\right]$$

$$\sim \frac{1}{5}R_2 \to R_2 \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 5 & -2 & 5 \\ 0 & 1 & 0 & -2 & 1 & -2 \\ 0 & 0 & 1 & -4 & 2 & -5 \end{array}\right]$$

$$\sim R_2+R_1 \to R_1 \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -1 & 3 \\ 0 & 1 & 0 & -2 & 1 & -2 \\ 0 & 0 & 1 & \underbrace{-4 \quad 2}_{A^{-1}} & & -5 \end{array}\right]$$

$$AX = B$$
$$A^{-1}AX = A^{-1}B$$
$$X = A^{-1}B$$

$$= \begin{bmatrix} 3 & -1 & 3 \\ -2 & 1 & -2 \\ -4 & 2 & -5 \end{bmatrix} \begin{bmatrix} 2 & -1 & 5 & 7 & 8 \\ 4 & 0 & -3 & 0 & 1 \\ 3 & 5 & -7 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 12 & -3 & 27 & 26 \\ -6 & -8 & 1 & -18 & -17 \\ -15 & -21 & 9 & -38 & -35 \end{bmatrix}$$

Question 2. §1.7 #TFI *(2 marks)* Determine whether the statement is true or false, and justify your answer.
If A^2 is a symmetric matrix, then A is a symmetric matrix.

False, Let $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$. A is not symmetric but $A^2 = AA = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ is symmetric

Question 3. §1.6 #33 *(3 marks)* Prove: If $A^TA = A$, then A is symmetric and $A = A^2$.

Premise:

$A^TA = A$

Conclusion:

- A is symmetric, $A^T = A$
- $A = A^2$

$LHS = A^T = (A^TA)^T = A^T(A^T)^T = A^TA = A = RHS$

$LHS = A = A^TA$

$= AA$ since A is symmetric

$= A^2$

$= RHS.$