Dawson College: Linear Algebra (SCIENCE): 201-NYC-05-S5: Winter 2017

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Quiz 6

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §2.2 #15 *(5 marks)* Evaluate the determinant of the given matrix by reducing the matrix to row echelon form or triangular form.

$$A = \begin{bmatrix} 2 & 1 & 3 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & 2 & 3 \end{bmatrix} \sim_{R_1 \leftrightarrow R_2} \begin{bmatrix} 1 & 0 & 1 & 1 \\ 2 & 1 & 3 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & 2 & 3 \end{bmatrix} \sim_{-2R_1 + R_2 \to R_2} \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & 2 & 3 \end{bmatrix}$$

$$\sim_{\substack{-2R_2 + R_3 \to R_3 \\ -R_2 + R_4 \to R_4}} \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

$$\sim_{R_3 + R_4 \to R_4} \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 6 \end{bmatrix} = B$$

(op. change det) det [orig. mat] = det [new mat]

$$(-1) \quad \det A = \det B$$

$$-\det A = -6$$

$$\det A = 6$$

Question 2. §2.1 #34 *(5 marks)* Show that the matrices

$$A = \begin{bmatrix} a & b \\ 0 & c \end{bmatrix} \text{ and } B = \begin{bmatrix} d & e \\ 0 & f \end{bmatrix}$$

commute if and only if

$$\begin{vmatrix} b & a-c \\ e & d-f \end{vmatrix} = 0$$

note: $AB = \begin{bmatrix} a & b \\ 0 & c \end{bmatrix}\begin{bmatrix} d & e \\ 0 & f \end{bmatrix} = \begin{bmatrix} ad & ae+bf \\ 0 & cf \end{bmatrix}$

$BA = \begin{bmatrix} d & e \\ 0 & f \end{bmatrix}\begin{bmatrix} a & b \\ 0 & c \end{bmatrix} = \begin{bmatrix} ad & bd+ce \\ 0 & cf \end{bmatrix}$

$\therefore$ A & B commute iff $ae + bf = bd + ce$

$ae - ce = bd - bf$

[⇒]

premise:

A & B commute

Conclusion: $\begin{vmatrix} b & a-c \\ e & d-f \end{vmatrix} = 0$

$LHS = \begin{vmatrix} b & a-c \\ e & d-f \end{vmatrix}$

$= b(d-f) - e(a-c)$

$= bd - bf - ea - ec$

$= ae - ce - ea - ec$ from premise

$= 0 = RHS$

[⇐] Premise: $\begin{vmatrix} b & a-c \\ e & d-f \end{vmatrix} = 0$

Conclusion: A & B commute

From the premise: $b(d-f) - e(a-c) = 0$

$bd - bf - ea + ec = 0$

$bd - bf = ae - ec$

$\therefore$ A & B commute.