

## Quiz 7

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

**Question 1.** §2.3 #21 (3 marks) Decide whether the given matrix is invertible, and if so, use the adjoint method to find its inverse.

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 0 & 1 & -3 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\det A = 2(1)(2) = 4 \neq 0 \quad \therefore A \text{ is invertible}$$

$$A^{-1} = \frac{1}{\det A} \text{adj} A$$

$$= \frac{1}{4} \begin{bmatrix} 2 & 0 & 0 \\ 6 & 4 & 0 \\ 4 & 6 & 2 \end{bmatrix}^T$$

$$= \frac{1}{4} \begin{bmatrix} 2 & 6 & 4 \\ 0 & 4 & 6 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\begin{array}{l} C_{11} = (-1)^{1+1} \begin{vmatrix} 1 & -3 \\ 0 & 2 \end{vmatrix} = 2 \\ C_{12} = (-1)^{1+2} \begin{vmatrix} 0 & -3 \\ 0 & 2 \end{vmatrix} = 0 \\ C_{13} = (-1)^{1+3} \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} = 0 \\ C_{21} = (-1)^{2+1} \begin{vmatrix} -3 & 5 \\ 0 & 2 \end{vmatrix} = 6 \\ C_{22} = (-1)^{2+2} \begin{vmatrix} 2 & 5 \\ 0 & 2 \end{vmatrix} = 4 \\ C_{23} = (-1)^{2+3} \begin{vmatrix} 2 & -3 \\ 0 & 0 \end{vmatrix} = 0 \\ C_{31} = (-1)^{3+1} \begin{vmatrix} -3 & 5 \\ 1 & -3 \end{vmatrix} = 4 \\ C_{32} = (-1)^{3+2} \begin{vmatrix} 2 & 5 \\ 0 & -3 \end{vmatrix} = 6 \\ C_{33} = (-1)^{3+3} \begin{vmatrix} 2 & -3 \\ 0 & 1 \end{vmatrix} = 2 \end{array}$$

**Question 2.** #3.4.10 (3 marks) Let  $B$  be a  $3 \times 3$  matrix where  $\det(B) = 3$ . Find  $\det(2B + B^2 \text{adj}(B))$ .

$$\det(2B + B^2 \text{adj}(B))$$

$$\text{note: } B^{-1} = \frac{1}{\det B} \text{adj} B$$

$$= \det(2B + B B^3 B^{-1})$$

$$= \det(2B + 3 B B B^{-1})$$

$$3 B^{-1} = \text{adj} B$$

$$= \det(2B + 3 B I)$$

$$= \det(5B)$$

$$= 5^3 \det B$$

$$= 5^3 \cdot 3$$

**Question 3.** §3.1 #TF (2 marks) Determine whether the statement is true or false, and justify your answer.

Two equivalent vectors must have the same initial point.

False, by definition 2 vectors are equivalent iff they have the same direction and magnitude. Their initial point is of no consequence.

**Question 4.** §3.1 #TF (2 marks) Determine whether the statement is true or false, and justify your answer.

If  $a$  and  $b$  are scalars such that  $a\vec{u} + b\vec{v} = \vec{0}$ , then  $\vec{u}$  and  $\vec{v}$  are parallel vectors.

False,

if  $a=b=0$  and  $\vec{u} = (1,1)$ ,  $\vec{v} = (1,2)$  then  $a\vec{u} + b\vec{v} = \vec{0}$  but  $\vec{u}$  and  $\vec{v}$  are not parallel vectors.