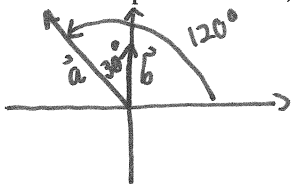
Name: Y. Lamontagne

Quiz 8

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §3.2 #15 *(3 marks)* Suppose that a vector $\vec{a}$ in the xy-plane has a length of 9 units and points in a direction that is 120° counterclockwise from the positive x-axis, and a vector $\vec{b}$ in that plane has a length of 5 units and points in the positive y-direction. Find $\vec{a}\cdot\vec{b}$.

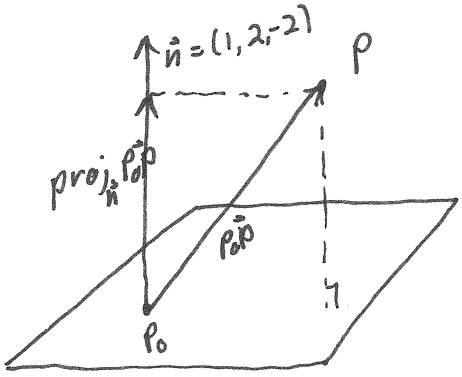


$\vec{a}\cdot\vec{b} = \|\vec{a}\|\|\vec{b}\|\cos\theta$

$\vec{a}\cdot\vec{b} = 9\cdot 5\cos 30°$

$\vec{a}\cdot\vec{b} = 45\,\frac{\sqrt{3}}{2}$

Question 2. §3.3 #33 *(4 marks)* Using projections find the distance between the point and the plane. $(3,\ 1,\ -2)$; $x+2y-2z=4$



Let $x=z=0 \Rightarrow y=2$ $\therefore P_0(0,2,0)$

$\vec{P_0P} = P - P_0 = (3,1,-2)-(0,2,0) = (3,-1,-2)$

$$\text{proj}_{\vec{n}}\vec{P_0P} = \frac{\vec{n}\cdot\vec{P_0P}}{\vec{n}\cdot\vec{n}}\vec{n}$$

$$= \frac{(1,2,-2)\cdot(3,-1,-2)}{(1,2,-2)\cdot(1,2,-2)}(1,2,-2)$$

$$= \frac{5}{9}(1,2,-2)$$

$$\text{distance} = \|\text{proj}_{\vec{n}}\vec{P_0P}\| = \left\|\frac{5}{9}(1,2,-2)\right\| = \frac{5}{9}\sqrt{1+4+4} = \frac{5\sqrt{9}}{9} = \frac{5\cdot 3}{9} = \frac{5}{3}$$

Question 3. #4.4.9 *(3 marks)* Determine the point of intersections (if any) for the pair of lines. $\vec{x}=(3,4,5)+t(1,1,1),\ t\in\mathbb{R}$ and $\vec{x}=(2,4,1)+s(2,3,-2),\ s\in\mathbb{R}$

From $\mathcal{L}_1$: $x=3+t$, $y=4+t$, $z=5+t$ and $\mathcal{L}_2$: $x=2+2s$, $y=4+3s$, $z=1-2s$. We obtain

$$\left.\begin{aligned} 3+t &= 2+2s \\ 4+t &= 4+3s \\ 5+t &= 1-2s \end{aligned}\right\} \Rightarrow \begin{aligned} 1 &= -t+2s \\ 0 &= -t+3s \\ 4 &= -t-2s \end{aligned}$$

$$\left[\begin{array}{cc|c} -1 & 2 & 1 \\ -1 & 3 & 0 \\ -1 & -2 & 4 \end{array}\right] \sim \begin{matrix} \\ -R_1+R_2\to R_2 \\ -R_1+R_3\to R_3 \end{matrix}\left[\begin{array}{cc|c} -1 & 2 & 1 \\ 0 & 1 & -1 \\ 0 & -4 & 3 \end{array}\right] \sim \begin{matrix} \\ \\ 4R_2+R_3\to R_3 \end{matrix}\left[\begin{array}{cc|c} -1 & 2 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & -1 \end{array}\right]$$

$\therefore$ the system is inconsistent

$\therefore$ the lines do not intersect.