Dawson College: Linear Algebra (SCIENCE): 201-NYC-05-S5: Winter 2017

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Quiz 9

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §3.4 #23

a. *(3 marks)* Find a homogeneous linear system of two equations in three unknowns whose solution space consists of those vectors in $\mathbb{R}^3$ that are othogonal to $\vec{a} = (1,1,1)$ and $\vec{b} = (-2,3,0)$.

b. *(2 marks)* What kind of geometric object is the solution space?

a) Let $\vec{v} = (x,y,z) \in \mathbb{R}^3$,

$\vec{v} \perp \vec{a} \iff \vec{v}\cdot\vec{a} = 0 \iff (x,y,z)\cdot(1,1,1) = 0$ ①

$\vec{v} \perp \vec{b} \iff \vec{v}\cdot b = 0 \iff (x,y,z)\cdot(-2,3,0) = 0$ ②

$\iff x+y+z = 0$

$\iff -2x+3y = 0$

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ -2 & 3 & 0 & 0 \end{bmatrix} \sim_{2R_1+R_2\to R_2} \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 5 & 2 & 0 \end{bmatrix} \sim_{\frac{1}{5}R_2\to R_2} \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 2/5 & 0 \end{bmatrix}$$

$$\sim_{-R_2+R_1\to R_1} \begin{bmatrix} 1 & 0 & 3/5 & 0 \\ 0 & 1 & 2/5 & 0 \end{bmatrix}$$

Let $z = t$ where $t \in \mathbb{R}$

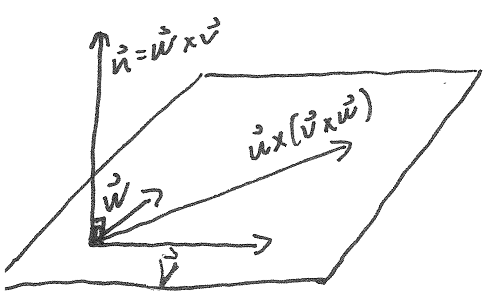
$x = -\frac{3}{5}t$

$y = -\frac{2}{5}t$

$\therefore (x,y,z) = t\left(-\frac{3}{5}, -\frac{2}{5}, 1\right)$

b) The solution set is a line since it is all multiples of a vector (the direction vector $\vec{d} = \left(\frac{-3}{5}, \frac{-2}{5}, 1\right)$).

Question 2. §3.5 #35 *(5 marks)* Show that if $\vec{u}, \vec{v}$ and $\vec{w}$ are vectors in $\mathbb{R}^3$, no two of which are collinear, then $\vec{u}\times(\vec{v}\times\vec{w})$ lies in the plane determined by $\vec{v}$ and $\vec{w}$.



$\vec{u}\times(\vec{v}\times\vec{w})$ lies on the plane determined by $\vec{v}$ and $\vec{w}$ iff $\vec{u}\times(\vec{v}\times\vec{w}) \perp \vec{n}$

iff $\vec{n}\cdot(\vec{u}\times(\vec{v}\times\vec{w})) = 0$

$\vec{n}\cdot(\vec{u}\times(\vec{v}\times\vec{w}))$

$= \vec{n}\cdot(\vec{u}\times\vec{n})$

$$= \begin{vmatrix} n_1 & n_2 & n_3 \\ u_1 & u_2 & u_3 \\ n_1 & n_2 & n_3 \end{vmatrix}$$

$= 0$ since $R_1 = R_3$

$\therefore \vec{u}\times(\vec{v}\times\vec{w})$ lies on the plane generated by $\vec{v}$ and $\vec{w}$.