Name: ____________________

Test 2

This test is graded out of 46 marks. No books, notes, watches or cell phones are allowed. You are only permitted to use the Sharp EL-531XG or Sharp EL-531X calculator. Give the work in full; – unless otherwise stated, reduce each answer to its simplest, exact form; – and write and arrange your exercise in a legible and orderly manner. If you need more space for your answer use the back of the page.

Question 1. Given

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -2 & -3 & -4 \\ 2 & 3 & 0 & 0 \\ 0 & 4 & 5 & 0 \end{bmatrix}.$$

a. *(5 marks)* Evaluate $\det(A)$.

b. *(5 marks)* If M is a 4×4 matrix such that $\det(M) = 2$ then evaluate $\det(\det(M)\mathrm{adj}(5M^T A^{-1}))$. Justify!

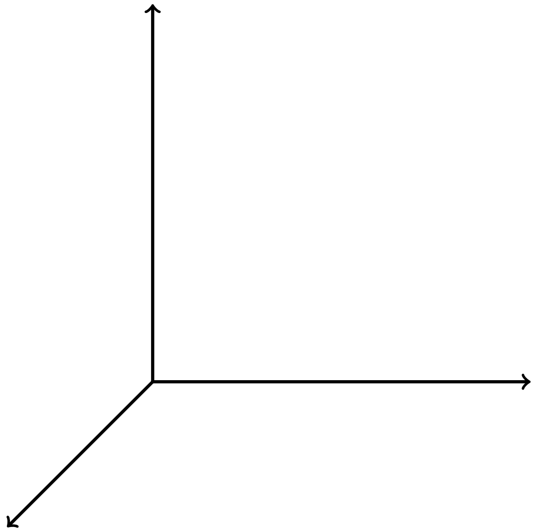
Question 2. *(3 marks)* Prove: If $AX = 0$ for some $X \neq 0$, then $\det(A) = 0$.

Question 3. *(3 marks)* Prove or disprove: There does not exist an $n \times n$ matrix A where n is odd such that $A^2 + I = 0$.

Question 4. *(3 marks)* Given $A = [a_{ij}]_{n\times n}$ and the cofactors of A, C_{ij}. Show that $a_{11}C_{21} + a_{12}C_{22} + \ldots + a_{1n}C_{2n} = 0$.

Question 5. Given the line $(x, y, z) = (1+3t, 2+2t, 3+t)$ where $t \in \mathbb{R}$ and point $P(3, 2, 1)$.

a. *(2 marks)* Sketch the given line using P_0 the point on the line when $t = 0$ and P_1 the point on the line when $t = 1$, also sketch P. Label the axes!



b. *(5 marks)* Using projections find area of the triangle defined by the points P_0, P_1 and P.

c. *(2 marks)* Find the equation of the line which passes through the point P and the closest point on the given line.

Question 6. *(4 marks)* Given $r \neq 0$ and $B = \begin{bmatrix} c & d \\ ra+2c & rb+2d \end{bmatrix}$ then express B as $E_3E_2E_1A$ where E_i are elementary matrices and $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. And given $\det(A) = 2$ find the determinant of B by using the expression $E_3E_2E_1A$.

Question 7. *(3 marks)* Prove: If $\vec{u}$ and $\vec{v}$ are orthogonal then $||\vec{u}+\vec{v}||^2 = ||\vec{u}||^2 + ||\vec{v}||^2$

Question 8. Given two planes $x+y+z=1$ and $x+2y+3z=4$.

a. *(2 marks)* Determine whether the two planes are perpendicular, parallel or neither. Justify!

b. *(3 marks)* Find the intersection between the two planes if it exists.

c. *(3 marks)* Find the angle between the normal of the two planes.

Bonus Question. *(5 marks)*
Prove: If A and B are two matrices such that $A+B=AB$ then A and B commute.