Name: ____________________

Test 3

This test is graded out of 41 marks. No books, notes, watches or cell phones are allowed. You are only permitted to use the Sharp EL-531XG or Sharp EL-531X calculator. Give the work in full; – unless otherwise stated, reduce each answer to its simplest, exact form; – and write and arrange your exercise in a legible and orderly manner. If you need more space for your answer use the back of the page.

Question 1. Given $\vec{u} = (-2,-1,0)$ and $\vec{v} = (1,0,1)$

$$\begin{array}{lcll} \mathscr{L}_1 & : & (x,y,z) = (1,0,1) & + \quad t_1\vec{u} \\ \mathscr{L}_2 & : & (x,y,z) = (-2,-1,2) & + \quad t_2\vec{v} \text{ where } t_1,\ t_2 \in \mathbb{R}. \end{array}$$

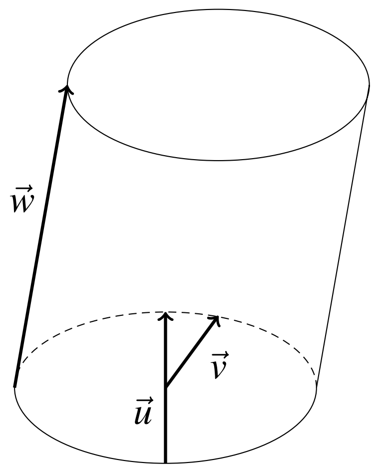
a. *(3 marks)* Determine whether the two lines intersect, are parallel or are skew lines.

b. *(3 marks)* Find the shortest distance between the two lines.

c. *(2 marks)* Find the area of the triangle defined by $\vec{u}$ and $\vec{v}$.

d. *(2 marks)* Find the equation of a line which is orthogonal to the direction of both given lines and passes through $\mathscr{L}_1$. Is the line unique?

Question 2. *(5 marks)* A cylinder where the sides are not perpendicular to the base (the circle) is called an *oblique cylinder*. Given the oblique cyclinder defined by the given vectors $\vec{u} = (2,2,4)$, $\vec{v} = (1,2,1)$ and $\vec{w} = (4,1,3)$. Note from the diagram below that $\vec{w}$ is not perpendicular to the base, that $\vec{v}$ is positioned such that its tail is at the center of the circle and its tip lies on the circle, that $\vec{u}$ is positioned such that the vector passes through the center of the circle while its tail and tip lie on the circle. Find the volume of the oblique cylinder. *(Hint: the volume of an oblique cyclinder is equal to the area of the base times the height.)*



Question 3. Given two planes:

$$
\begin{array}{rcrcrcrcr}
\mathscr{P}_1 & : & x & - & 2y & + & z & = & 1 \\
\mathscr{P}_2 & : & -4x & + & 8y & - & 4z & = & -4
\end{array}
$$

a. *(1 mark)* Give a geometrical argument to explain why the intersection of the two planes is a plane.

b. *(2 marks)* Find the solution set of the associated homogeneous linear system.

c. *(2 marks)* Show that the solution set of the associated homegeneous linear system is orthogonal to the rows of the coefficient matrix of the system.

d. *(2 marks)* Give a geometrical interpretation to part c).

Question 4. A function f is said to be *odd* if $f(-x) = -f(x)$. Let $V = \{f \mid f : \mathbb{R} \to \mathbb{R} \text{ and } f \text{ is an odd function}\}$ with the following vector addition and scalar multiplication:

$$(f+g)(x) = f(x)g(x) \text{ and } (kf)(x) = (f(x))^k$$

a. *(3 marks)* Is the zero vector an element of V? Justify.

b. *(3 marks)* Determine whether the following axiom holds: $(rs)f = r(sf)$ where $r,\ s \in \mathbb{R}$ and $f \in V$

Question 5. *(3 marks)* Determine whether $W = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \mid (ab)^2 = (cd)^2 \right\}$ is a subspace of $\mathscr{M}_{2\times 2}$.

Question 6. Given $\vec{u} = (1,2,3), \vec{v} = (-3,-6,\lambda) \in \mathbb{R}^3$ and the set $S = \{\vec{u},\ \vec{v},\ \vec{u} \times \vec{v}\}$.

a. *(2 marks)* Find the value(s), if any, of λ for which S spans $\mathbb{R}^3$

b. *(2 marks)* Find the value(s), if any, of λ for which S spans a plane.

c. *(2 marks)* Find the value(s), if any, of λ for which S spans a line.

Question 7. Given $W = \left\{a_0 + a_1 x + a_2 x^2 + a_3 x^3 \mid a_0 + 2a_1 + 3a_2 + 4a_3 = 0 \text{ and } a_1 + a_2 + a_3 = 0\right\}$ a subspace of P_3.

a. *(3 marks)* Find a basis B for W.

b. *(1 mark)* State the $\dim(W)$ and $\dim(P_3)$.

Bonus Question. *from Wikipedia (3 marks)* Discuss the following: The barber is the "one who shaves all those, and those only, who do not shave themselves." The question is, does the barber shave himself?