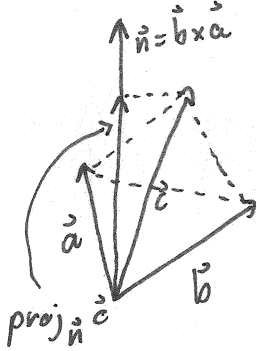


## Quiz 10

This quiz is graded out of 8 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

**Question 1.** §3.5 #38 It is a theorem of solid geometry that the volume of a tetrahedron is  $\frac{1}{3}$ (area of base) · (height). Use this result to prove that the volume of a tetrahedron whose sides are the vectors  $\vec{a}$ ,  $\vec{b}$ , and  $\vec{c}$  is  $\frac{1}{6}|\vec{a} \cdot (\vec{b} \times \vec{c})|$  (see accompanying figure).



$$\begin{aligned}
 V &= \frac{1}{3} (\text{area of base}) \cdot (\text{height}) \\
 &= \frac{1}{3} \frac{1}{2} \|\vec{b} \times \vec{a}\| \|\text{proj}_{\vec{n}} \vec{c}\| \\
 &= \frac{1}{6} \|\vec{n}\| \left\| \frac{\vec{c} \cdot \vec{n}}{\vec{n} \cdot \vec{n}} \vec{n} \right\| \\
 &= \frac{1}{6} \frac{\|\vec{n}\| |\vec{c} \cdot \vec{n}| \|\vec{n}\|}{\|\vec{n}\|^2} \\
 &= \frac{1}{6} |\vec{c} \cdot (\vec{b} \times \vec{a})| \\
 &= \frac{1}{6} \begin{vmatrix} c_1 & c_2 & c_3 \\ b_1 & b_2 & b_3 \\ a_1 & a_2 & a_3 \end{vmatrix} \\
 &= \frac{1}{6} \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \\
 &= \frac{1}{6} \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \\
 &= \frac{1}{6} |\vec{a} \cdot (\vec{b} \times \vec{c})|
 \end{aligned}$$