Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S2: Winter 2018

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Quiz 11

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §4.1 #38 Show that the set of all points in $\mathbb{R}^2$ lying on a line is a vector space with respect to the standard operations of vector addition and scalar multiplication if and only if the line passes through the origin. Let $V = \{(x,y) \mid ax+by=c\}$

[$\Rightarrow$] premise: V is a vector space

conclusion: $c=0$

Suppose $c \neq 0$ (proof by contradiction). Since V is a vector space it must be closed under addition. Let $(x_1,y_1),(x_2,y_2) \in V$

$(x_1,y_1)+(x_2,y_2) = (x_1+x_2, y_1+y_2) \notin V$ since $a(x_1+x_2)+b(y_1+y_2)$

$= ax_1+ax_2+by_1+by_2$

$= ax_1+by_1+ax_2+by_2$

$= c+c$

$= 2c \neq 0$ since $c \neq 0$

$\therefore c=0$

[$\Leftarrow$] premise: $V = \{(x,y) \mid ax+by=0\}$

conclusion: V is a vector space

① closure under addition: Let $(x_1,y_1),(x_2,y_2) \in V$, $(x_1,y_1)+(x_2,y_2) = (x_1+x_2, y_1+y_2) \in V$ since $a(x_1+x_2)+b(y_1+y_2) = ax_1+by_1+ax_2+by_2 = 0+0=0$ since $(x_1,y_1),(x_2,y_2) \in V$

④ zero vector: Let $(x,y) \in V$ and $\vec{0} = (a,b)$, $(x,y)+(a,b) = (x,y)$, $(x+a, y+b) = (x,y) \Rightarrow (a,b) = (0,0)$

$\vec{0} = (0,0) \in V$ since $a(0)+b(0) = 0$.

⑤ additive inverse: Let $(x,y) \in V$ and $\vec{w} = (s,t)$ $(x,y)+(s,t) = (0,0) \Rightarrow (s,t) = (-x,-y)$

$\vec{w} \in V$ since $a(-x)+b(-y) = -(ax+by) = -(0) = 0$ since $(x,y) \in V$

⑥ closure under scalar multiplication: Let $(x,y) \in V$ and $r \in \mathbb{R}$, $r(x,y) = (rx,ry) \in V$ since $a(rx)+b(ry) = arx+bry = r(ax+by) = r(0) = 0$ since $(x,y) \in V$

all other axioms hold since V is a subset of $\mathbb{R}^2$ and those properties hold in $\mathbb{R}^2$.