Dawson College: Linear Algebra (SCIENCE): 201-NYC-05-S2: Winter 2018

Name: Y. Lamontagne

Quiz 12

This quiz is graded out of 6 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §4.2 #2g Determine whether the following is subspaces of $\mathscr{M}_{n\times n}$. The set of all $n\times n$ matrices A such that $AB = BA$ for some fixed $n\times n$ matrix B.

Let $V = \{A \mid A \in M_{n\times n}$ and $AB = BA$ where B is fixed$\}$

Lets apply the subspace test

① closure under addition

Let $M_1, M_2 \in V$ $\quad M_1 + M_2 \in V$ since $(M_1+M_2)B = M_1B + M_2B$

$= BM_1 + BM_2$ since $M_1, M_2 \in V$

$= B(M_1+M_2)$

$\therefore$ closed under vector addition

② closure under scalar multiplication

Let $M \in V$ and $r \in \mathbb{R}$, $\quad rM \in V$ since $(rM)B = rMB$

$= rBM$ since $M \in V$

$= BrM = B(rM)$

$\therefore$ closed under scalar multiplication

$\therefore$ V is a subspace of $M_{2\times 2}$.

Question 2. §4.3 #28 Prove that in $\mathscr{P}_2$ every set with more than three vectors is linearly dependent.

Let $S = \{\vec{p}_1, \vec{p}_2, \dots, \vec{p}_n\}$ where $n > 3$ and $\vec{p}_i = a_{i0} + a_{i1}x + a_{i2}x^2$

$$c_1\vec{p}_1 + c_2\vec{p}_2 + \dots + c_n\vec{p}_n = \vec{0}$$

$$\vec{0} = c_1(a_{10} + a_{11}x + a_{12}x^2) + c_2(a_{20} + a_{21}x + a_{22}x^2) + \dots + c_n(a_{n0} + a_{n1}x + a_{n2}x^2)$$

$$0 + 0x + 0x^2 = (c_1a_{10} + c_2a_{20} + \dots + c_na_{n0}) + (c_1a_{11} + c_2a_{21} + \dots c_na_{n1})x + (c_1a_{12} + c_2a_{22} + \dots + c_na_{n2})x^2$$

$$\begin{bmatrix} a_{10} & a_{20} & \dots & a_{n0} \\ a_{11} & a_{21} & \dots & a_{n1} \\ a_{12} & a_{22} & \dots & a_{n2} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ \vdots \\ c_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

The homogeneous system has an infinite number of solution since #var > #equation when $n > 3$.
Hence not only the trivial solution.
$\therefore$ S is linearly dependent.

$$\left[\begin{array}{cccc|c} a_{10} & a_{20} & \dots & a_{n0} & 0 \\ a_{11} & a_{21} & \dots & a_{n1} & 0 \\ a_{12} & a_{22} & \dots & a_{n2} & 0 \end{array}\right]$$