

Quiz 3

This quiz is graded out of 6 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §1.3 #36

a. Prove: If AB and BA are both defined, then AB and BA are square matrices.

Let A be a $m \times n$ matrix and B be a $p \times q$ matrix.

premise: AB and BA are both defined.

conclusion: AB and BA are square matrices.

Since AB is defined then # cols of $A =$ # rows of B , hence $n = p$
and AB is an $m \times q$ matrix.

Since BA is defined then # cols of $B =$ # rows of A , hence $m = q$
and BA is an $p \times n$ matrix. By $*$ we have that BA is an $n \times n$ matrix.

Since AB is an $m \times q$ matrix and by $**$ AB is an $m \times m$ matrix.

\therefore AB and BA are square matrices.

b. Prove: If A is an $m \times n$ matrix and $A(BA)$ is defined, then B is an $n \times m$ matrix.

Let B be a $p \times q$ matrix.

premise: A is $m \times n$ matrix, $A(BA)$ is defined

conclusion: B is an $n \times m$ matrix

In order for $A(BA)$ to be defined, BA needs to be defined. In order for BA to be defined $q = m$, and BA is a $p \times n$ matrix.

Since $A(BA)$ is defined # col of $A =$ # row of BA , hence $p = n$

\therefore B is an $n \times m$ matrix by $*$ and $**$