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Quiz 4

This quiz is graded out of 6 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §1.4 #33

a. Show that if a square matrix A satisfies the equation $A^2 + 2A + I = 0$, then A must be invertible. What is the inverse?

premise:

- A is a square matrix
- $A^2 + 2A + I = 0$

conclusion:

- A is invertible

$$A^2 + 2A + I = 0$$
$$I = -2A - A^2$$
$$I = A(-2I - A) \text{ and } I = (-2I - A)A$$

$\therefore$ A is invertible and $A^{-1} = (-2I - A)$

b. Show that if $p(x)$ is a polynomial with a nonzero constant term, and if A is a square matrix for which $p(A) = 0$, then A is invertible.

premise:

- A is a square matrix
- $p(x) = a_m x^m + \dots + a_2 x^2 + a_1 x + a_0$ where $a_0 \neq 0$ and $p(A) = 0$

conclusion:

- A is invertible

$$p(A) = a_m A^m + \dots + a_2 A^2 + a_1 A + a_0 I = 0$$
$$a_0 I = -a_1 A - a_2 A^2 - \dots - a_m A^m$$
$$I = -\frac{a_1}{a_0} A - \frac{a_2}{a_0} A^2 - \dots - \frac{a_m}{a_0} A^m \quad \text{since } a_0 \neq 0$$

$$I = A\left(-\frac{a_1}{a_0} I - \frac{a_2}{a_0} A - \dots - \frac{a_m}{a_0} A^{m-1}\right) \text{ and } I = \left(-\frac{a_1}{a_0} I - \frac{a_2}{a_0} A - \dots - \frac{a_m}{a_0} A^{m-1}\right) A$$

$\therefore$ A is invertible and $A^{-1} = -\frac{a_1}{a_0} I - \frac{a_2}{a_0} A - \dots - \frac{a_m}{a_0} A^{m-1}$