Dawson College: Linear Algebra (SCIENCE): 201-NYC-05-S2: Winter 2018

Name: Y. Lamontagne

Quiz 5

This quiz is graded out of 6 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §1.5 #TF In each parts, determine whether the statement is true or false, and justify your answer.

a. The product of two elementary matrices of the same size must be an elementary matrix.

False,

$I \sim_{\pi R_1 \to R_1} \begin{bmatrix} \pi & 0 \\ 0 & 1 \end{bmatrix} = E_1$

$I \sim_{\pi R_2 \to R_2} \begin{bmatrix} 1 & 0 \\ 0 & \pi \end{bmatrix} = E_2$

but $E_1 E_2 = \begin{bmatrix} \pi & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & \pi \end{bmatrix} = \begin{bmatrix} \pi & 0 \\ 0 & \pi \end{bmatrix} = A$

E_1 and E_2 are elementary matrices but A is not since it can not be obtained by a single elementary row operation on I.

f. If A is invertible and a multiple of the first row of A is added to the second row, then the resulting matrix is invertible.

True,

$A \sim_{kR_1 + R_2 \to R_2} B$ from the premise, from the elementary row operation we obtain $I \sim_{kR_1 + R_2 \to R_2} E$ its associated elementary matrix such that $EA = B$. B is invertible since it can be expressed as a product of invertible matrices.

g. An expression of an invertible matrix A as a product of elementary matrices is unique.

False, let $A = \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix}$

$A \sim_{R_1 \leftrightarrow R_2} \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \sim_{\frac{1}{2}R_1 \to R_1} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$E_2 E_1 A = I$

$A = E_1^{-1} E_2^{-1}$

$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$

$A \sim_{\frac{1}{2}R_2 \to R_2} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \sim_{R_1 \leftrightarrow R_2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$E_2 E_1 A = I$

$A = E_1^{-1} E_2^{-1}$

$A = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

Two different expression of A as a product of elementary matrices